

Cambridge International AS & A Level

CANDIDATE NAME		
CENTRE NUMBER		CANDIDATE NUMBER
FURTHER MA	ATHEMATICS	9231/1



Paper 1 Further Pure Mathematics 1

May/June 2022

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer all questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do not use an erasable pen or correction fluid.
- Do not write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working plearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has 20 pages. Any blank pages are indicated.

1 Let a be a positive constant.

(a) Use the method of differences to find
$$\sum_{r=1}^{n} \frac{1}{(ar+1)(ar+a+1)}$$
 in terms of n and a . [4]

(b) Find the value of
$$a$$
 for which
$$\sum_{r=1}^{\infty} \frac{1}{(ar+1)(ar+a+1)} = \frac{1}{6}.$$
 [3]

 $\frac{1}{a^2 t a} = \frac{1}{6}$

 $a^2 + a - 6 = 0$ $a^2 + 3a - 2a - 6 = 0$

 $(\alpha+3)(\alpha-2)=0$

a=2

2 The points A, B, C have position vectors

$$4i-4j+k$$
, $-4i+3j-4k$, $4i-j-2k$,

respectively, relative to the origin O.

(a)	Find the equation of the pla $\overrightarrow{OR} = 4i - 4j + k$	the ABC, giving your answer in the form $ax + by + cz = d$.	[5]
	0B = -4i+3j-4K		
	0C = 4i-j-2k	$\overrightarrow{AB} = -8i + 7j - 5k$	
	AC = OC - OA = 41/	-j-2K-4i+4j-k	
	AC = 31-314	J	

$$\begin{vmatrix}
 i & j & k \\
 i & -8 & 7 & -5 \\
 0 & 3 & -3
 \end{vmatrix}
 = i \begin{vmatrix} 7 & -5 \\ 3 & -3 \end{vmatrix} - j \begin{vmatrix} -8 & -5 \\ 0 & -3 \end{vmatrix} + k \begin{vmatrix} -8 & 7 \\ 0 & 3 \end{vmatrix}$$

$$= i(-21+15) - j(24) + K(-24)$$

$$= -6i - 24j - 24k \implies \begin{pmatrix} -6 \\ -24 \\ -24 \end{pmatrix} \sim \begin{pmatrix} 4 \\ 4 \\ \end{pmatrix}$$

$$1 + 4y + 4z = d$$

$$d = 4 + 4(-4) + 4(1) = -8$$

2+44	+ 42= -8				
<i>J</i>					
			 	•••••	•••••
 		•••••	 		•••••
 	•••••	• • • • • • • • • • • • • • • • • • • •	 •••••	•••••	• • • • • • • • • • • • • • • • • • • •

(b)	Find the perpendicular distance from <i>O</i> to the plane <i>ABC</i> .	[2
	8 = 1.39	
	$\sqrt{1^2 + 4^2 + 4^2}$	
(c)	The point <i>D</i> has position vector $2\mathbf{i} + 3\mathbf{j} - 3\mathbf{k}$. Find the coordinates of the point of intersection of the line <i>OD</i> with the plane <i>ABC</i> .	[3
	$\Gamma = \left(\begin{array}{c} 2\\ 3\\ -3 \end{array}\right) = \left(\begin{array}{c} 2t\\ 3b\\ -3t \end{array}\right)$	
	n+4y+42=-8	
	2t + 4(3t) + 4(-3t) = -8	
	2t+126-126=-& t=-4	
	8 \	
	$\Rightarrow \begin{pmatrix} -i2 \\ i2 \end{pmatrix}$	
		••••

3	The sequence of positive numbers u_1, u_2, u_3, \dots	ic cuch that u	> 1 and for $n > 1$
J	The sequence of positive numbers u_1, u_2, u_2, \dots	. Is such that u_1	/ T and, for $n > 1$

$$u_{n+1} = \frac{u_n^2 + u_n + 12}{2u_n}.$$

for n=	<u>- [</u>
	4,74 as given
Suppose	true for n=k
	UK+1 = UK2 + UK + 12
	2u _K
Prove 1	rue for n=k+1
	$u_{k+1} - 4 = u_k^2 + u_k + 12 - 4$
	24/6
	4KH-4= 4K2+4K+12-8uge
	2u _k
	$y_{k+1} - y = y_k^2 - 7y_k + 12$
	242
	$4k_{+1} - 4 = 4k^2 - 44k - 34k + 12$
	24 _{IC}
	$y_{1c+1}-4=(y_k-4)(y_k-3)$
	2u <u>k</u>
	$(u_{k}-4)(u_{k}-3) > 6$
	24 _E
	y ₁₀₊₁ - 4 >0
	U _{C+1} 7 4
1.1	ence induction is complete.

Show that $u_{n+1} < u_n$ for $n \ge 1$.	
un2+ un+12 < Nn	
2 ₁ / ₀	
$u_{\underline{n}} + u_{\underline{n}} + 12 = u_{\underline{n}} < 0$	
2u,	
-un2+un+12 20	
~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~	
$-(u_{N}-4)(u_{N}+3) < 0$	
24 _n	
u, >4	
$(u_{n}-4)(u_{n}+3) > 0$	$-u_1 - u_2 < 0$
24 ₁	u _{n+1} < u _n
v\	"N+1 ""N
- (u - u) ( 2)	
$-(u_{n}-4)(u_{n}+3)<0$	

4	The cubic equation	$2x^3 + 5x^2 - 6 = 0$	has roots	α, β, γ.
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(a) Find a cubic equation whose roots are  $\frac{1}{\alpha^3}$ ,  $\frac{1}{\beta^3}$ ,  $\frac{1}{\gamma^3}$ .

 $y = \frac{1}{x^3} = 0 x^{-3} = 0 x = y^{-1/3}$ 

[3]

 $2y' + 5y^{-\frac{2}{3}} - 6 = 0 = 1$  5y' = 6 - 2y'

 $125y = (6y - 2)^3$ 

 $216y^3 - 216y^2 - 53y - 8 = 0$ 

(b) Find the value of  $\frac{1}{\alpha^6} + \frac{1}{\beta^6} + \frac{1}{\gamma^6}$ . [3]  $\frac{1}{\gamma^6} + \frac{1}{\beta^6} + \frac{1}{\gamma^6} = \left(\frac{1}{\gamma^3}\right)^2 + \left(\frac{1}{\beta^3}\right)^2 + \left(\frac{1}{\gamma^3}\right)^2$ 

 $= \left( \frac{\bot + \bot}{9^3} + \frac{\bot}{7^3} \right) - 2 \left( \frac{\bot}{9^3 \beta^3}, \frac{\bot}{9^3 \gamma^3} + \frac{\bot}{\beta^3 \gamma^3} \right)$ 

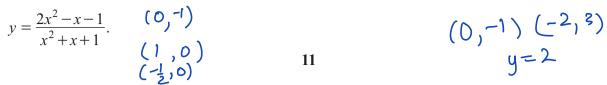
 $= (1)^{2} - 2(-53)$  = 216

 $\frac{1}{9^6} + \frac{1}{9^6} + \frac{1}{108} = \frac{161}{108}$ 

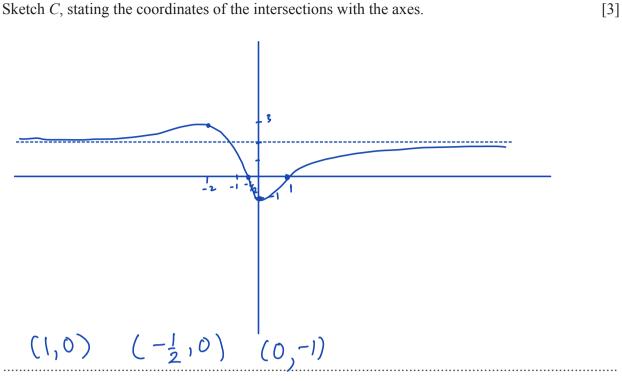
1 1 1	
Find also the value of $\frac{1}{10^9} + \frac{1}{10^9} + \frac{1}{10^9}$ .	[2]
$\alpha$ $\beta$ $\gamma$	
$\alpha \beta \gamma$	C(.3
	$\left(\frac{1}{2}\right)^3$
Find also the value of $\frac{1}{\alpha^9} + \frac{1}{\beta^9} + \frac{1}{\gamma^9}$ . $\frac{1}{\gamma^9} + \frac{1}{\beta^9} + \frac{1}{\gamma^9} = \left(\frac{1}{\beta^3}\right) + \left(\frac{1}{\beta^3}\right) + \frac{1}{\beta^9} + \frac{1}{\gamma^9} = \left(\frac{1}{\beta^3}\right) + \frac{1}{\beta^9} = 1$	( <u>/</u> 3)
$216y^3 - 216y^2 - 53y - 8 = 0$ $216S_3 - 216S_2 - 53S_7 - 24 = 0$	)
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	)
$216y^{3} - 216y^{2} - 53y - 8 = 0$ $216 C_{3} - 216 C_{2} - 53C_{3} - 24 = 0$ $C_{2} = 216 \left(\frac{161}{108}\right) + 53(1) = 0$	)
$216y^{3} - 216y^{2} - 53y - 8 = 0$ $216 S_{3} - 216S_{2} - 53S_{1} - 24 = 0$	)
$216y^{3} - 216y^{2} - 53y - 8 = 0$ $216 \cdot S_{3} - 216S_{2} - 53S_{3} - 24 = 0$ $S_{2} = 216 \left(\frac{161}{108}\right) + 53(1) = 0$	)
$216y^{3} - 216y^{2} - 53y - 8 = 0$ $216S_{3} - 216S_{2} - 53S_{1} - 24 = 0$ $S_{3} = 216\left(\frac{161}{108}\right) + 53(1) = 0$ $216$	)
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The curve $C$ has equation	<i>y</i> =	$\frac{2x^2-x-1}{x^2+x+1}$
	The curve $C$ has equation	The curve $C$ has equation $y =$

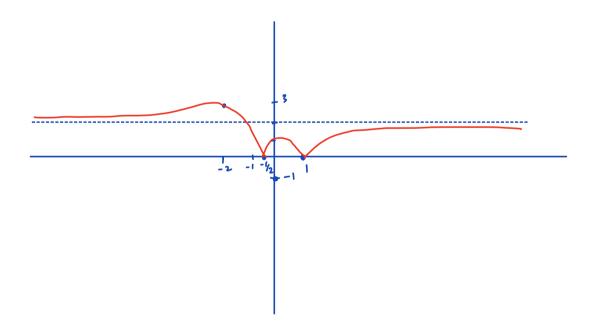
(a)	Show that C	has no vertica	l asymptotes a	and state the	equation	on of the	e horizonta	al asymptote of <i>C</i> . [3]
	2+2+	1=0		y=2	<u> </u>	HA		
	b2-40	1C < 6			J 			
	12-4(1	)(1) <0						
	1-4	20						
	-3 4(	)						
	so no re	al solo						
	so no	VAs.						
(b)	Find the coore							[4]
	dy = (2	2+ n+1) (4°	(2-1)	12+1)(22	² -	-1)		
	dn		(x2+x+1	)2				
	3x2+6x	= D						
	32(2+2	-)=0						
	7=0	21+2	=D					
	y	1 x=	-2					
		y -	- 3					
	(0,	-1) (	-2,3)					



(c) Sketch C, stating the coordinates of the intersections with the axes.

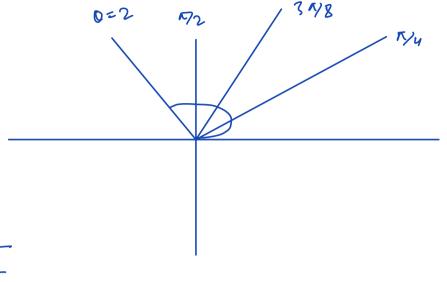


(d) Sketch the curve with equation  $y = \left| \frac{2x^2 - x - 1}{x^2 + x + 1} \right|$  and state the set of values of k for which  $\left| \frac{2x^2 - x - 1}{x^2 + x + 1} \right| = k$  has 4 distinct real solutions. [2]



02K21

- 6 The curve C has polar equation  $r^2 = \tan^{-1}(\frac{1}{2}\theta)$ , where  $0 \le \theta \le 2$ .
  - (a) Sketch C and state, in exact form, the greatest distance of a point on C from the pole. [3]



**(b)** Find the exact value of the area of the region bounded by C and the half-line  $\theta = 2$ . [5]

= (2) d0	Lu du an	= uv-1	v dy dz
6	du		dn
2			

$$\frac{1}{2} \left[ 0 \tan^{-1} \left( \frac{1}{2} 0 \right) - \int_{0}^{2} \frac{1}{2} 0 \cdot \frac{1}{2} \right] \qquad u = \tan^{-1} \left( \frac{1}{2} 0 \right) \qquad v' = 1$$

$$\frac{1}{2} \left[ 0 \tan^{-1} \left( \frac{1}{2} 0 \right) - \int_{0}^{2} \frac{20}{0^{2} + 1} d d d \right] \qquad u' = \frac{1}{2} \cdot \frac{1}{2} \qquad v = 0$$

$$\frac{1}{2} \left[ 0 \tan^{-1} \left( \frac{1}{2} 0 \right) - \ln \left( 0^2 + 11 \right) \right]_0^2$$

1/5- 2 lu8+ 2 lu4=	1/2 - 1/2 LM2	
	7	

ow consider the part of $C$ where $0 \le \theta \le \frac{1}{2}\pi$ .  Show that, at the point furthest from the half-line $\theta = \frac{1}{2}\pi$ , $(\theta^2 + 4) \tan^{-1} \left(\frac{1}{2}\theta\right) \sin \theta - \cos \theta = 0$ and verify that this equation has a root between 0.6 and 0.7. $C = \tan^{-1} \left(\frac{1}{2}\theta\right)$ $\pi = \left[\tan^{-1} \left(\frac{1}{2}\theta\right)\right]^{\frac{1}{2}} \cos \theta$ $\pi = \int_{0}^{\infty} \tan^{-1} \left(\frac{1}{2}\theta\right) \int_{0}^{\infty} \cos \theta = 0$ $\tan^{-1} \left(\frac{1}{2}\theta\right) \int_{0}^{\infty} \cos \theta = 0$ $\tan^{-1} \left(\frac{1}{2}\theta\right) \int_{0}^{\infty} \cos \theta = 0$ $\tan^{-1} \left(\frac{1}{2}\theta\right) \int_{0}^{\infty} \cos \theta = 0$ $\cos^{-1} \left(\frac{1}{2}\theta\right) \sin \theta - \cos \theta = 0$ $\cos^{-1} \left(\frac{1}{2}\theta\right) \sin \theta - \cos \theta = 0$ $\cos^{-1} \left(\frac{1}{2}\theta\right) \sin \theta - \cos \theta = 0$ $\cos^{-1} \left(\frac{1}{2}\theta\right) \sin \theta - \cos \theta = 0$ $\cos^{-1} \left(\frac{1}{2}\theta\right) \sin \theta - \cos \theta = 0$ $\cos^{-1} \left(\frac{1}{2}\theta\right) \sin \theta - \cos \theta = 0$ $\cos^{-1} \left(\frac{1}{2}\theta\right) \sin \theta - \cos \theta = 0$ $\cos^{-1} \left(\frac{1}{2}\theta\right) \sin \theta - \cos \theta = 0$ $\cos^{-1} \left(\frac{1}{2}\theta\right) \sin \theta - \cos \theta = 0$ $\cos^{-1} \left(\frac{1}{2}\theta\right) \sin \theta - \cos \theta = 0$ $\cos^{-1} \left(\frac{1}{2}\theta\right) \sin \theta - \cos \theta = 0$ $\cos^{-1} \left(\frac{1}{2}\theta\right) \sin \theta - \cos \theta = 0$ $\cos^{-1} \left(\frac{1}{2}\theta\right) \cos \theta + \cos^{-1} \left(\frac{1}{2}\theta$		
Show that, at the point furthest from the half-line $\theta = \frac{1}{2}\pi$ , $(\theta^2 + 4) \tan^{-1} \left(\frac{1}{2}\theta\right) \sin \theta - \cos \theta = 0$ and verify that this equation has a root between 0.6 and 0.7. $(\frac{2}{2} - \tan^{-1} \left(\frac{1}{2}\theta\right)) \frac{1}{2} \cos \theta$ $(\frac{1}{2} - \sin \theta) \left(\tan^{-1} \left(\frac{1}{2}\theta\right)\right) \frac{1}{2} \cos \theta$ $(\frac{1}{2} - \sin \theta) \left(\tan^{-1} \left(\frac{1}{2}\theta\right)\right) \frac{1}{2} \cos \theta$ $(\frac{1}{2} - \sin \theta) \left(\tan^{-1} \left(\frac{1}{2}\theta\right)\right) \frac{1}{2} \cos \theta$ $(\frac{1}{2} - \sin \theta) \left(\tan^{-1} \left(\frac{1}{2}\theta\right)\right) \sin \theta - \cos \theta = 0$ $(\frac{1}{2} + \frac{1}{2} - \sin \theta) \left(\tan^{-1} \left(\frac{1}{2}\theta\right)\right) \sin \theta - \cos \theta = 0$ $(\frac{1}{2} - \sin \theta) \left(\tan^{-1} \left(\frac{1}{2}\theta\right)\right) \sin \theta - \cos \theta = 0$ $(\frac{1}{2} - \sin \theta) \left(\tan^{-1} \left(\frac{1}{2}\theta\right)\right) \sin \theta - \cos \theta = 0$		
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Show that, at the point furthest from the half-line $\theta = \frac{1}{2}\pi$ , $(\theta^2 + 4) \tan^{-1} \left(\frac{1}{2}\theta\right) \sin \theta - \cos \theta = 0$ and verify that this equation has a root between 0.6 and 0.7. $c^2 = \tan^{-1} \left(\frac{1}{2}\theta\right)$ $\pi = \left[\tan^{-1} \left(\frac{1}{2}\theta\right)\right]^{\frac{1}{2}} \cos \theta$ $\pi = \int_{-\infty}^{\infty} \tan^{-1} \left(\frac{1}{2}\theta\right) \int_{-\infty}^{\infty} \cos \theta$ $\frac{1}{2} $	we consider the part of Cychoro $0 < 0 < \frac{1}{2}\pi$	
and verify that this equation has a root between 0.6 and 0.7. $ \begin{aligned} r^2 &= tam^{-1} \left(\frac{1}{2}\theta\right) & \pi = r\cos\theta \\ \pi &= \left[tam^{-1} \left(\frac{1}{2}\theta\right)\right]^{\frac{1}{2}} \cos\theta & r &= \pi \\ d\pi &= -\sin\theta \left(tam^{-1} \left(\frac{1}{2}\theta\right)\right)^{\frac{1}{2}} + \cos\theta \left(tam^{-1} \left(\frac{1}{2}\theta\right)\right) \cdot \left(\theta^2 + 4\right) = 0 \end{aligned} $ $ \frac{d\theta}{d\theta} = -\sin\theta \left(tam^{-1} \left(\frac{1}{2}\theta\right)\right)^{\frac{1}{2}} + \cos\theta \left(tam^{-1} \left(\frac{1}{2}\theta\right)\right) \cdot \left(\theta^2 + 4\right) = 0 $ $ \frac{d\theta}{d\theta} = -\cos\theta \left(tam^{-1} \left(\frac{1}{2}\theta\right)\right) \cdot (\theta^2 + 4) = 0 $ $ \frac{d\theta}{d\theta} = -\cos\theta \left(tam^{-1} \left(\frac{1}{2}\theta\right)\right) \cdot (\theta^2 + 4) = 0 $ $ \frac{d\theta}{d\theta} = -\cos\theta = 0 $		
and verify that this equation has a root between 0.6 and 0.7. $ \begin{aligned} & r^2 = \tan^{-1}\left(\frac{1}{2}\theta\right) & \chi = r\cos\theta \\ & \chi = \left[\tan^{-1}\left(\frac{1}{2}\theta\right)\right]^{\frac{1}{2}}\cos\theta & r = \frac{\pi}{\cos\theta} \\ & d\eta = -\sin\theta \left(\tan^{-1}\left(\frac{1}{2}\theta\right)\right)^{\frac{1}{2}} + \cos\theta \left(\tan^{-1}\left(\frac{1}{2}\theta\right)\right)^{\frac{1}{2}} \cdot \left(\theta^2 + 4\right)^{\frac{1}{2}} = 0 \end{aligned} $ $ \frac{d\eta}{d\theta} = -\sin\theta \left(\tan^{-1}\left(\frac{1}{2}\theta\right)\right)^{\frac{1}{2}} + \cos\theta \left(\tan^{-1}\left(\frac{1}{2}\theta\right)\right)^{\frac{1}{2}} \cdot \left(\theta^2 + 4\right)^{\frac{1}{2}} = 0 $ $ \frac{d\theta}{d\theta} = -\cos\theta \left(\tan^{-1}\left(\frac{1}{2}\theta\right)\right)^{\frac{1}{2}} + \cos\theta \left(\tan^{-1}\left(\frac{1}{2}\theta\right)\right)^{\frac{1}{2}} \cdot \left(\theta^2 + 4\right)^{\frac{1}{2}} = 0 $ $ \frac{d\theta}{d\theta} = -\cos\theta \left(\tan^{-1}\left(\frac{1}{2}\theta\right)\right)^{\frac{1}{2}} + \cos\theta \left(\tan^{-1}\left(\frac{1}{2}\theta\right)\right)^{\frac{1}{2}} \cdot \left(\theta^2 + 4\right)^{\frac{1}{2}} = 0 $ $ \frac{d\theta}{d\theta} = -\cos\theta \left(\tan^{-1}\left(\frac{1}{2}\theta\right)\right)^{\frac{1}{2}} + \cos\theta \left(\tan^{-1}\left(\frac{1}{2}\theta\right)\right)^{\frac{1}{2}} \cdot \left(\theta^2 + 4\right)^{\frac{1}{2}} = 0 $ $ \frac{d\theta}{d\theta} = -\cos\theta \left(\tan\theta\right) + \cos\theta \left$	Show that, at the point furthest from the half-line $\theta = \frac{1}{2}\pi$ ,	
	$(\theta^2 + 4)\tan^{-1}\left(\frac{1}{2}\theta\right)\sin\theta - \cos\theta = 0$	
$\frac{d1}{d0} = -\sin\theta \left( \tan^{-1} \left( \frac{1}{2} 0 \right) \right)^{\frac{1}{2}} + \cos\theta \left( \tan^{-1} \left( \frac{1}{2} 0 \right) \right)^{\frac{1}{2}} \cdot \left( \delta^{2} + 4 \right)^{\frac{1}{2}} = 0$ $\frac{d0}{(0^{2} + 4) \tan^{-1} \left( \frac{1}{2} 0 \right) \sin\theta - \cos\theta} = 0$ $\frac{d0}{(0^{2} + 4) \tan^{-1} \left( \frac{1}{2} 0 \right) \sin\theta - \cos\theta} = 0$ $\frac{d0}{(0^{2} + 4) \tan^{-1} \left( \frac{1}{2} 0 \right) \sin\theta - \cos\theta} = 0$	and verify that this equation has a root between 0.6 and 0.7.	]
$\frac{d1}{d0} = -\sin\theta \left( \tan^{-1} \left( \frac{1}{2} 0 \right) \right)^{\frac{1}{2}} + \cos\theta \left( \tan^{-1} \left( \frac{1}{2} 0 \right) \right)^{\frac{1}{2}} \cdot \left( \delta^{2} + 4 \right)^{\frac{1}{2}} = 0$ $(0^{2} + 4) \tan^{-1} \left( \frac{1}{2} 0 \right) \sin\theta - \cos\theta = 0$ $\int_{0}^{\infty} 0 \cdot 6 = 0 - 0.10 $ $\int_{0}^{\infty} 0 \cdot 7 = 0.20 $	$c^2 = \tan^{-1}(\frac{1}{2}0)$	7=rcos0
$\frac{d7}{d0} = -\sin\theta \left( \tan^{-1} \left( \frac{1}{2} \theta \right) \right)^{\frac{1}{2}} + \cos\theta \left( \tan^{-1} \left( \frac{1}{2} \theta \right) \right)^{\frac{1}{2}} \cdot \left( \theta^{2} + 4 \right)^{\frac{1}{2}} = 0$ $(\theta^{2} + 4) \tan^{-1} \left( \frac{1}{2} \theta \right) \sin\theta - \cos\theta = 0$ $(\theta^{2} + 4) \tan^{-1} \left( \frac{1}{2} \theta \right) \sin\theta - \cos\theta = 0$ $(\theta^{2} + 4) \tan^{-1} \left( \frac{1}{2} \theta \right) \sin\theta - \cos\theta = 0$ $(\theta^{2} + 4) \tan^{-1} \left( \frac{1}{2} \theta \right) \sin\theta - \cos\theta = 0$	$y = \left[ \tan^{-1} \left( \frac{1}{2} Q \right) \right]^{2} \cos Q$	r= 7
$(0^{2}+4) \tan^{-1}(\frac{1}{2}0) \sin 0 - \cos 0 = 0$ $for 0.6 = 0 - 0.103$ $for 0.7 = 0.209$		0050
$(0^{2}+4)\tan^{-1}(\frac{1}{20})\sin \theta - \cos \theta = 0$ $for 0.6 = 0 - 0.10$ $for 0.7 = 0.209$	-1.1/2	7/2 (2)
$ (0^{2}+4) tom^{-1}(\frac{1}{20}) sin0 - cos0 = 0 $ $ for 0.6 = 0 - 0.103 $ $ for 0.7 = 0.209 $	$dr = -\sin\theta(\tan^2(20)) + \cos\theta(\tan^2(20))$	$) \cdot (0+4) = 0$
for 0.6 =) - 0.108 for 0.7 = 0.209	do	
for 0.6 =) - 0.103 for 0.7 = 0.209	$(0^2 + 4)$ from $(\frac{1}{2}0)$ cino - caso = 0	
V	(017)(000 (20)31110 3000	
V		
V	1010.6=) - 0.108	
V	HOY 0.7 = 0.209	
Sign change so coot exists 11/w them	V	
Sign Change so look exists n/w Them	010.10	٠
	Sign Change so loot exists	n/w Them

		1	2	3	١
7	The matrix $\mathbf{A}$ is given by $\mathbf{A} = \begin{bmatrix} \mathbf{A} & \mathbf{A} \\ \mathbf{A} & \mathbf{A} \end{bmatrix}$	4	k	6	١
		7	8	9	

(a)	Find the set of values of $k$ for which $A$ is non-singular.	[3]
	$  \begin{array}{c}   \begin{array}{c}   \\   \\   \\   \\   \\   \\   \\   \\   \\   $	
	1 8 9 1 T 1 T 1 T 1 T 1 T 1 T 1 T 1 T 1 T 1	••••••
	-12K+60=0	
	K 75 (K <5, K >5)	
	$R \neq J$ $(F \subseteq J, E \neq J)$	
(b)	Given that <b>A</b> is non-singular, find, in terms of $k$ , the entries in the top row of $\mathbf{A}^{-1}$ .	[4]
	$C_{11} =   \frac{K}{8}   \frac{6}{9}   = 9K - 48$	
		•••••
	$C_{12} = -\begin{vmatrix} 2 & 3 \\ 8 & 9 \end{vmatrix} = -(18 - 24) = 6$	
	$c_{13} = \begin{vmatrix} 2 & 3 \\ k & 6 \end{vmatrix} = 12 - 3k$	
	~13 - 1 K 6 1 - 1/2 - 1	
	$A = \begin{pmatrix} 9k - 48 & \frac{6}{60 - 12k} & \frac{12 - 31k}{60 - 2k} \\ -\frac{12 - 31k}{60 - 12k} & \frac{60 - 2k}{60 - 2k} \end{pmatrix}$	
	0-12k 60-12k 60-2k	•••••
	M = ( )	
		•••••
		•••••

c)	Given that $\mathbf{B} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$ , give an example of a matrix $\mathbf{C}$ such that $\mathbf{BAC} = \begin{pmatrix} 2 & 1 \\ k & 4 \end{pmatrix}$ . [4]
	$A = \begin{pmatrix} 1 & 23 \\ 4 & 6 \end{pmatrix}$ $789$
	$BA = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 \\ 4 & 1 & 6 \\ 7 & 8 & 9 \end{pmatrix}$
	$BA = \begin{pmatrix} 1 & 2 & 3 \\ 4 & K & 6 \end{pmatrix}$
	$\begin{pmatrix} 1 & 23 \\ 4 & 6 \end{pmatrix} \begin{pmatrix} a & q \\ b & e \\ c & f \end{pmatrix} = \begin{pmatrix} a+2b+3c & d+2e+3f \\ 4a+kb+6c & 4d+ke+6f \end{pmatrix} = \begin{pmatrix} 2 & 1 \\ kq & 4d+ke+6f \end{pmatrix}$
	let $b=1$ , $a=0$ , $c=0$ $e=0$ , $f=0$ , $d=1$

(d)	Find the set of values of $k$ for which the transformation in the $x$ - $y$ plane represented by $\begin{pmatrix} 2 & 1 \\ k & 4 \end{pmatrix}$ has two distinct invariant lines through the origin. [6] $\begin{pmatrix} 2 & 1 \\ k & 4 \end{pmatrix} \begin{pmatrix} 3 \\ 1 \end{pmatrix} = \begin{pmatrix} 2n + y \\ 2n + 4y \end{pmatrix}$
	Cn+4mn=m(2n+mn)
	$m^2 - 2m - 1c = 0$
	4+4k >0
	K>-1

## Additional page

If you use the following page to complete the answer to any questic shown.	on, the question number must be clearly

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