Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

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2064332093

FURTHER MATHEMATICS **9231/31**

Paper 3 Further Mechanics **October/November 2022**

1 hour 30 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.
- Where a numerical value for the acceleration due to gravity (g) is needed, use $10\,\text{m}\,\text{s}^{-2}$.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Any blank pages are indicated.

1 A particle of mass 2 kg is attached to one end of a light inextensible string of length 0.6 m. The other end of the string is attached to a fixed point on a smooth horizontal surface. The particle is moving in a circular path on the surface. The tension in the string is 20 N.

Find how many revolutions the particle makes per minute. [3]

2 A light elastic string has natural length a and modulus of elasticity $4mg$. One end of the string is fixed to a point O on a smooth horizontal surface. A particle P of mass m is attached to the other end of the string. The particle P is projected along the surface in the direction OP. When the length of the string is $\frac{5}{4}a$, the speed of P is v. When the length of the string is $\frac{3}{2}a$, the speed of P is $\frac{1}{2}v$.

(a) Find an expression for v in terms of a and g. [4]

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(b) Find, in terms of g, the acceleration of P when the stretched length of the string is $\frac{3}{2}a$. [2]

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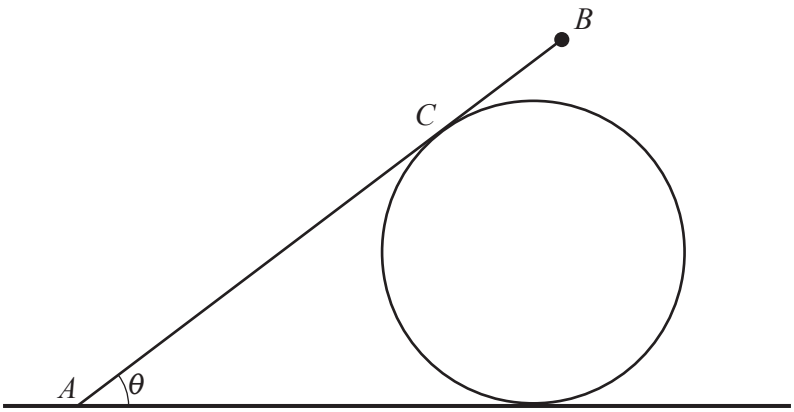

A smooth cylinder is fixed to a rough horizontal surface with its axis of symmetry horizontal. A uniform rod AB, of length $4a$ and weight W, rests against the surface of the cylinder. The end A of the rod is in contact with the horizontal surface. The vertical plane containing the rod AB is perpendicular to the axis of the cylinder. The point of contact between the rod and the cylinder is C, where $AC = 3a$. The angle between the rod and the horizontal surface is θ where $\tan\theta = \frac{3}{4}$ (see diagram). The coefficient of friction between the rod and the horizontal surface is $\frac{6}{7}$.

A particle of weight kW is attached to the rod at B. The rod is about to slip. The normal reaction between the rod and the cylinder is N.

(a) Show that $N = \frac{8}{15}W(1+2k)$. [2]

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(b) Find the value of k. [5]

4 A particle of mass 0.5 kg moves along a horizontal straight line. Its velocity is $v\,\text{m s}^{-1}$ at time t s. The forces acting on the particle are a driving force of magnitude 50 N and a resistance of magnitude $2v^2$ N. The initial velocity of the particle is $3\,\text{m s}^{-1}$.

(a) Find an expression for v in terms of t. [7]

(b) Deduce the limiting value of v. [1]

5 A particle P of mass m is attached to one end of a light inextensible string of length a. The other end of the string is attached to a fixed point O. The string is held taut with OP horizontal. The particle P is projected vertically downwards with speed $\sqrt{\frac{1}{3}ag}$ and starts to move in a vertical circle. P passes through the lowest point of the circle and reaches the point Q where OQ makes an angle θ with the downward vertical. At Q the speed of P is $\sqrt{kag}$ and the tension in the string is $\frac{11}{6}mg$.

(a) Find the value of k and the value of $\cos\theta$. [4]

At Q the particle P becomes detached from the string.

(b) In the subsequent motion, find the greatest height reached by P above the level of the lowest point of the circle. [4]

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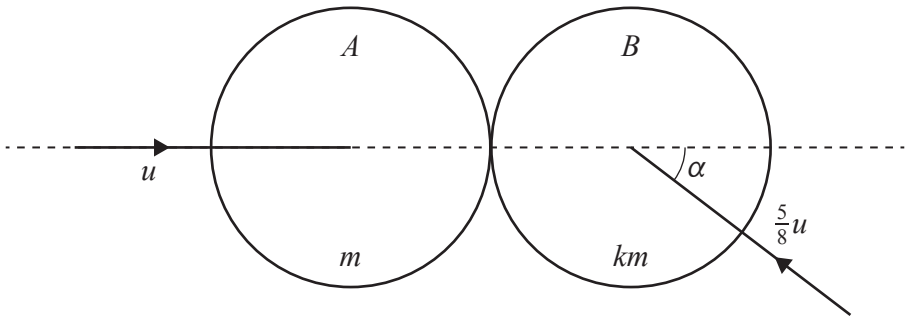

Two uniform smooth spheres A and B of equal radii have masses m and km respectively. The two spheres are moving on a horizontal surface with speeds u and $\frac{5}{8}u$ respectively. Immediately before the spheres collide, A is travelling along the line of centres, and B's direction of motion makes an angle α with the line of centres (see diagram). The coefficient of restitution between the spheres is $\frac{2}{3}$ and $\tan\alpha = \frac{3}{4}$.

After the collision, the direction of motion of B is perpendicular to the line of centres.

(a) Find the value of k. [4]

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(b) Find the loss in the total kinetic energy as a result of the collision. [4]

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7 A particle P is projected with speed $V\,\text{m}\,\text{s}^{-1}$ at an angle 75° above the horizontal from a point O on a horizontal plane. It then moves freely under gravity.

(a) Show that the total time of flight, in seconds, is $\frac{2V}{g}\sin 75°$. [2]

A smooth vertical barrier is now inserted with its lower end on the plane at a distance 15 m from O. The particle is projected as before but now strikes the barrier, rebounds and returns to O. The coefficient of restitution between the barrier and the particle is $\frac{3}{5}$.

(b) Explain why the total time of flight is unchanged. [1]

(c) Find an expression for V in terms of g. [7]

Additional page

If you use the following page to complete the answer to any question, the question number must be clearly shown.

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FURTHER MATHEMATICS **9231/31**

Paper 3 Further Mechanics **October/November 2022**

MARK SCHEME

Maximum Mark: 50

Published

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PUBLISHED

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:
Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:
Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:
Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:
Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

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Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	Use F = ma: $20 = \frac{2 \times v^2}{0.6}$ OR $20 = 2 \times 0.6\omega^2$	**M1**	
	$v^2 = 6$ OR $\omega^2 = \frac{50}{3}$	**A1**	
	Number of revolutions per min = $\frac{60v}{0.6 \times 2\pi}$ OR $\frac{60\omega}{2\pi}$ so 39(.0) revolutions	**A1 FT**	38.9848….
		3	

Question	Answer	Marks	Guidance
2(a)	Loss in KE = Gain in EPE, so $\frac{1}{2}mv^2 - \frac{1}{2}m\left(\frac{v}{2}\right)^2 = \frac{1}{2} \times \frac{4mg}{a}\left(\left(\frac{1}{2}a\right)^2 - \left(\frac{1}{4}a\right)^2\right)$	**B1**	EPE terms correct.
		M1	All 4 terms and no extras.
	$\frac{3}{4}mv^2 = \frac{4mg}{a} \times \frac{3}{16}a^2$	**M1**	Simplify.
	$v^2 = ag, \quad v = \sqrt{ag}$	**A1**	
		4	
2(b)	Hooke's law: tension = $\frac{4mg}{a} \times \frac{1}{2}a$ (= $2mg$)	**M1**	
	Acceleration = $\frac{2mg}{m} = 2g$	**A1**	Accept -2*g*.
		2	

Question	Answer	Marks	Guidance
3(a)	Let F and R be friction and normal reaction at A Take moments about A, for rod $N\times 3a = W\times 2a\cos\theta + kW\times 4a\cos\theta$	**M1**	Correct terms, allow sign errors and cos/sin mix.
	$3N = (2+4k)W\times\frac{4}{5}$ $N = \frac{8}{15}W(1+2k)$	**A1**	At least one intermediate line of working. AG
		2	
3(b)	$\uparrow N\cos\theta + R = W + kW$	**B1**	Resolve (to include R) for rod.
	$\rightarrow F = N\sin\theta$ and $F = \frac{6}{7}R$	**B1**	Both.
	so $R = \frac{28}{75}W(1+2k)$ or $R = \frac{21}{45}W(1+k)$	**M1**	Find R or N.
	Eliminate to find k	**M1**	Complete method.
	$k = \frac{1}{3}$	**A1**	
		5	

Question	Answer	Marks	Guidance
4(a)	$m\frac{dv}{dt} = 50 - 2v^2 \quad \frac{dv}{dt} = 4\left(25 - v^2\right)$	**B1**	N2L
	$\frac{1}{10}\int \frac{1}{5-v} + \frac{1}{5+v}\,dv = \int 4\,dt$	**M1**	Separate variables and use partial fractions.
	$\frac{1}{10}\left(-\ln\left(5-v\right) + \ln\left(5+v\right)\right) = 4t + A$	**M1 A1**	Integrate into log terms. (Note: formula on MF19).
	Use $t = 0, v = 3$ to give $A = \frac{1}{10}\ln 4$	**M1**	Use initial condition.
	$4t = \frac{1}{10}\ln\frac{5+v}{4\left(5-v\right)}$ leading to $\frac{5+v}{20-4v} = \mathrm{e}^{40t}$	**M1**	Rearrange to make v the subject.
	$v = \frac{5\left(4 - \mathrm{e}^{-40t}\right)}{4 + \mathrm{e}^{-40t}}$	**A1**	
		7	
4(b)	As $t \to \infty, v \to 5$	**B1**	
		1	

Question	Answer	Marks	Guidance
5(a)	$\frac{1}{2}mv^2 - \frac{1}{2}mu^2 = mga\cos\theta$ $\left[kag = \frac{1}{3}ag + 2ag\cos\theta\right]$	**B1**	Energy equation.
	$T - mg\cos\theta = \frac{m}{a}v^2$ So $\frac{11}{6}mg - mg\cos\theta = \frac{m}{a}.kag$, $\frac{11}{6} - \cos\theta = k$	**B1**	N2L at B.
	Solve simultaneously.	**M1**	
	$k = \frac{4}{3}$, $\cos\theta = \frac{1}{2}$	**A1**	Both.
		4	
5(b)	Initial speed $\uparrow = \sqrt{kag}\sin\theta$	**B1**	
	Use $v^2 = u^2 + 2as$: $0 = \left(\sqrt{kag}\sin\theta\right)^2 - 2gs$	**M1**	
	$s = \frac{1}{2}a$	**A1**	
	Height above lowest point = $s + a - a\cos\theta = \frac{1}{2}a + a - \frac{1}{2}a = a$	**A1 FT**	
		4	

Question	Answer	Marks	Guidance
6(a)	Let speed of A after collision be $\rightarrow v_A$ and speed of B perpendicular to line of centres be $\downarrow v$ Along line of centres: $mu - km\frac{5}{8}u\cos\alpha = mv_A$	**M1**	Momentum.
	NEL: $0 - v_A = e\left(\frac{5}{8}u\cos\alpha + u\right)$	**M1**	NEL
	So $u - \frac{5}{8}ku\cos\alpha = -\frac{2}{3}\left(\frac{5}{8}u\cos\alpha + u\right)$	**M1**	Solve.
	Substitute for cos, to give $k = 4$	**A1**	
		4	
6(b)	$v_B = \frac{5}{8}u\sin\alpha = \frac{3}{8}u$	**B1**	Velocity perpendicular to line of centres
	$v_A = -u$	**B1 FT**	
	KE before = $\frac{1}{2}mu^2 + \frac{1}{2}km\left(\frac{5}{8}u\right)^2 = \frac{1}{2}mu^2 + \frac{25}{32}mu^2 = \frac{41}{32}mu^2$ KE after = $\frac{1}{2}mv_A{}^2 + \frac{1}{2}kmv_B{}^2 = \frac{1}{2}mu^2 + 2m\frac{9}{64}u^2 = \frac{25}{32}mu^2$	**M1**	NOTE: KE before and after for *A* is unchanged. Both.
	Loss = $mu^2\left(\frac{41}{32} - \frac{25}{32}\right) = \frac{1}{2}mu^2$	**A1**	
		4	

Question	Answer	Marks	Guidance
7(a)	$\uparrow 0 = V\sin 75t - \frac{1}{2}gt^2$	**M1**	
	$t = \frac{2V}{g}\sin 75°$	**A1**	AG
		2	
7(b)	Vertical component of velocity is unchanged.	**B1**	
		1	

Question	Answer	Marks	Guidance
7(c)	Horizontally to wall, $\rightarrow 15 = V\cos 75t; \left(t = \frac{15}{V\cos 75}\right)$	**B1**	
	eVcos75	**B1**	Speed after rebound.
	$T = \frac{15}{eV\cos 75}$	**M1**	Time back to O $\left(t = \frac{3}{5}T\right)$
	Vertically for whole flight: $t + T = \frac{2V}{g}\sin 75°$	**M1**	
	$\frac{15}{V\cos 75} + \frac{15}{eV\cos 75} = \frac{2V}{g}\sin 75°$	**A1**	
	$V^2 \cos 75 \sin 75 = 20g$	**M1**	
	Multiply by 2: $V^2 \sin 150 = 40g, \quad V^2 = 80g$ $V = 4\sqrt{5g}\left(= 8.94\sqrt{g}\right)$	**A1**	
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1 hour 30 minutes

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1 A particle P of mass m is attached to one end of a light inextensible string of length a. The other end of the string is attached to a fixed point O. The string is held taut with OP making an angle α with the downward vertical, where $\cos\alpha = \frac{2}{3}$. The particle P is projected perpendicular to OP in an upwards direction with speed $\sqrt{3ag}$. It then starts to move along a circular path in a vertical plane.

Find the cosine of the angle between the string and the upward vertical when the string first becomes slack. [4]

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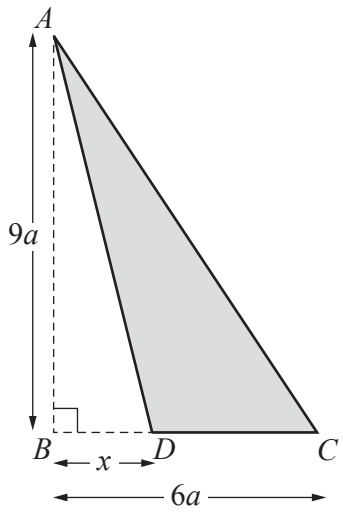

A uniform lamina is in the form of a triangle ABC in which angle B is a right angle, $AB = 9a$ and $BC = 6a$. The point D is on BC such that $BD = x$ (see diagram). The region ABD is removed from the lamina. The resulting shape ADC is placed with the edge DC on a horizontal surface and the plane ADC is vertical.

Find the set of values of x, in terms of a, for which the shape is in equilibrium. [6]

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3 One end of a light elastic string, of natural length a and modulus of elasticity $\frac{16}{3}Mg$, is attached to a fixed point O. A particle P of mass $4M$ is attached to the other end of the string and hangs vertically in equilibrium. Another particle of mass $2M$ is attached to P and the combined particle is then released from rest. The speed of the combined particle when it has descended a distance $\frac{1}{4}a$ is v.

Find an expression for v in terms of g and a. [6]

4 A particle P of mass 5 kg moves along a horizontal straight line. At time t s, the velocity of P is v m s^{-1} and its displacement from a fixed point O on the line is x m. The forces acting on P are a force of magnitude $\frac{500}{v}$ N in the direction OP and a resistive force of magnitude $\frac{1}{2}v^2$ N. When $t = 0$, $x = 0$ and $v = 5$.

(a) Find an expression for v in terms of x. [6]

(b) State the value that the speed approaches for large values of x. [1]

5 A particle P is projected with speed $u\,\text{m s}^{-1}$ at an angle of θ above the horizontal from a point O on a horizontal plane and moves freely under gravity. The horizontal and vertical displacements of P from O at a subsequent time t s are denoted by x m and y m respectively.

(a) Show that the equation of the trajectory is given by

$$y = x\tan\theta - \frac{gx^2}{2u^2}(1+\tan^2\theta).$$ [4]

In the subsequent motion P passes through the point with coordinates (30, 20).

(b) Given that one possible value of $\tan\theta$ is $\frac{4}{3}$, find the other possible value of $\tan\theta$. [5]

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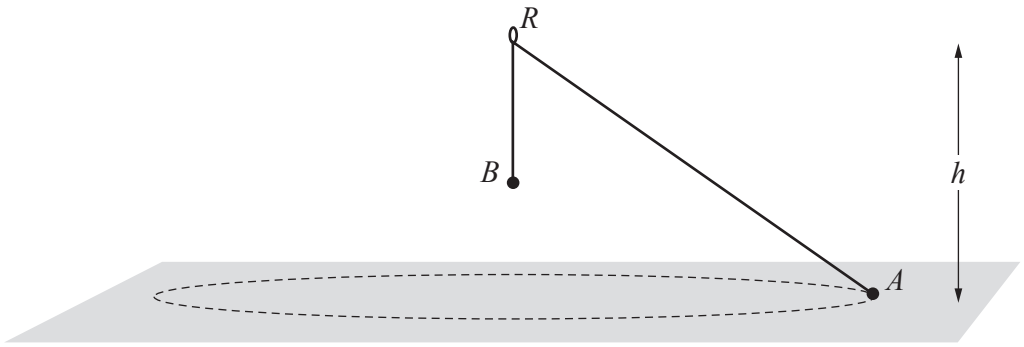

A light inextensible string is threaded through a fixed smooth ring R which is at a height h above a smooth horizontal surface. One end of the string is attached to a particle A of mass m. The other end of the string is attached to a particle B of mass $\frac{6}{7}m$. The particle A moves in a horizontal circle on the surface. The particle B hangs in equilibrium below the ring and above the surface (see diagram).

When A has constant angular speed ω, the angle between AR and BR is θ and the normal reaction between A and the surface is N.

When A has constant angular speed $\frac{3}{2}\omega$, the angle between AR and BR is α and the normal reaction between A and the surface is $\frac{1}{2}N$.

(a) Show that $\cos\theta = \frac{4}{9}\cos\alpha$. [5]

(b) Find N in terms of m and g and find the value of $\cos\alpha$. [4]

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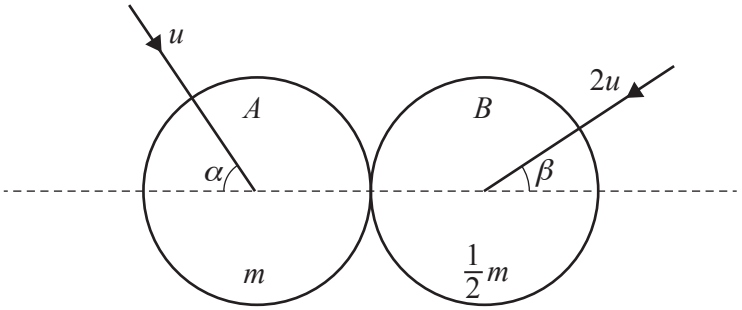

Two uniform smooth spheres A and B of equal radii have masses m and $\frac{1}{2}m$ respectively. The two spheres are moving on a horizontal surface when they collide. Immediately before the collision, sphere A is travelling with speed u and its direction of motion makes an angle α with the line of centres. Sphere B is travelling with speed $2u$ and its direction of motion makes an angle β with the line of centres (see diagram). The coefficient of restitution between the spheres is $\frac{5}{8}$ and $\alpha + \beta = 90°$.

(a) Find the component of the velocity of B parallel to the line of centres after the collision, giving your answer in terms of u and α. [4]

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The direction of motion of B after the collision is parallel to the direction of motion of A before the collision.

(b) Find the value of $\tan\alpha$. [5]

Additional page

If you use the following page to complete the answer to any question, the question number must be clearly shown.

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Types of mark

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WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	When string goes slack, $mg\cos\beta = \frac{m}{a}v^2$, $v^2 = ag\cos\beta$	**B1**	N2L May include T, but B1 not awarded until $T = 0$.
	$\frac{1}{2}m.3ag - \frac{1}{2}mv^2 = mg\left(a\cos\alpha + a\cos\beta\right)$	**B1**	Energy equation.
	So $u^2 - ag\cos\beta = 2ag\left(\cos\beta + \frac{2}{3}\right)$	**M1**	Combine.
	$\cos\beta = \frac{u^2 - \frac{4}{3}ag}{3ag} = \frac{5}{9}$	**A1**	
		4	

Question	Answer	Marks	Guidance
2	<table><tr><th></th><th>Area</th><th>Distance from AB</th></tr><tr><td>ABC</td><td>$27a^2$</td><td>$2a$</td></tr><tr><td>ABD</td><td>$\frac{9}{2}ax$</td><td>$\frac{1}{3}x$</td></tr><tr><td>Shape ADC</td><td>$27a^2-\frac{9}{2}ax$</td><td>$\bar{x}$</td></tr></table>		
	Taking moments about AB $\bar{x}\times\left(27a^2-\frac{9}{2}ax\right)=27a^2\times 2a-\frac{9}{2}ax\times\frac{1}{3}x$ $\left[\bar{x}=\frac{54a^3-\frac{3}{2}ax^2}{27a^2-\frac{9}{2}ax}\right]$	**M1** **A1** **A1**	Moments equation with 3 terms. At least 2 terms correct. All correct.
	For equilibrium, $x\leqslant\bar{x}$, $54a^3-\frac{3}{2}ax^2\geqslant x\left(27a^2-\frac{9}{2}ax\right)$	**B1**	Use correct condition: allow strict inequality. Can be implied by correct final answer $x\leqslant 3a$.
	$54a^2-27ax+3x^2\geqslant 0$ $(x-3a)(x-6a)\geqslant 0$	**M1**	Simplify and attempt to solve a quadratic inequality or equation.
	$(0\leqslant)\ x\leqslant 3a$ [only]	**A1**	CAO

Question	Answer	Marks	Guidance
2	**Alternative method for question 2**		
	Taking moments with B as origin.	**M1**	
	$\bar{x} = \frac{1}{3}(0 + x + 6a) = 2a + \frac{1}{3}x$	**A2**	
	For equilibrium, $x \leqslant \bar{x}$, so $x \leqslant 2a + \frac{1}{3}x$	**B1**	Allow strict inequality.
	$(0 \leqslant)\ x \leqslant 3a$	**M1**	
		A1	
		6	

Question	Answer	Marks	Guidance
3	In equilibrium, $\frac{\frac{16}{3}Mge}{a} = 4Mg, \quad e = \frac{3}{4}a$	**B1**	
	In subsequent motion, Loss in GPE = gain in EPE + gain in KE	**M1**	Energy equation with GPE and KE terms correct and at least one EPE term. Dimensionally correct.
	$\frac{6Mga}{4} = \frac{1}{2}.\frac{16}{3}.\frac{Mg}{a}.\left(a^2 - \left(\frac{3a}{4}\right)^2\right) + \frac{1}{2}.6Mv^2$	**B1**	EPE correct.
		A1	All correct.
	$\frac{3Mga}{2} = \frac{8Mg}{3a}.\frac{7}{16}a^2 + 3Mv^2$ etc	**M1**	Attempt to find v in terms of a and g.
	$\frac{ga}{3} = 3v^2, \quad v = \frac{1}{3}\sqrt{ga}$	**A1**	
		6	

Question	Answer	Marks	Guidance
4(a)	$5v\frac{dv}{dx}=\frac{500}{v}-\frac{1}{2}v^2$	**B1**	Sight of m or 5 is required.
	$\frac{10v^2dv}{1000-v^3}=dx$	**M1**	Separate variables and attempt to integrate into a log term.
	$-\frac{10}{3}\ln\left(1000-v^3\right)=x\left(+A\right)$	**A1**	
	$x=0, v=5, \quad A=-\frac{10}{3}\ln 875$	**M1**	Evaluate constant: correct initial condition used.
	$x=\frac{10}{3}\ln\frac{875}{1000-v^3}$	**M1**	Make v the subject: correct use of logs.
	$v=\left[(1000-875e^{-0.3x}\right]^{\frac{1}{3}}$	**A1**	$v=5\left[(8-7e^{-0.3x}\right]^{\frac{1}{3}}$: A0 if $e^{\ln}$ terms.
		6	
4(b)	Maximum value of v is 10	**B1**	No FT: result can be found from initial equation.
		1	

Question	Answer	Marks	Guidance
5(a)	$\rightarrow x = u\cos\theta t$	**B1**	Result quoted from MF19 scores 0/4.
	$\uparrow y = u\sin\theta t - \frac{1}{2}gt^2$	**B1**	
	Eliminate t: $y = u\sin\theta \times \frac{x}{\cos\theta} - \frac{1}{2}g\left(\frac{x}{u\cos\theta}\right)^2$	**M1**	
	$y = x\tan\theta - \frac{gx^2}{2u^2}(1+\tan^2\theta)$	**A1**	Must be an intermediate line of working. AG
		4	
5(b)	$20 = 30\times\frac{4}{3} - 10\times\frac{30^2}{2u^2}\times\left(1+\left(\frac{4}{3}\right)^2\right)$	**M1**	Substituting values correctly.
	$u^2 = 625,\ [u = 25]$	**A1**	
	Substitute back into trajectory equation, $20 = 30\tan\theta - \frac{g30^2}{2.25^2}\sec^2\theta = 30\tan\theta - \frac{36}{5}(1+\tan^2\theta)$ $18\tan^2\theta - 75\tan\theta + 68 = 0$	**M1**	Obtain a 3-term quadratic.
	One solution is $\frac{4}{3}$, $(3\tan\theta - 4)(6\tan\theta - 17) = 0$	**M1**	
	Giving $\tan\theta = \frac{17}{6}$	**A1**	
		5	

Question	Answer	Marks	Guidance
6(a)	$T = \frac{6}{7}mg$	**B1**	May be implied.
	$T\sin\theta = mr\omega^2 = mh\tan\theta \times \omega^2$	**B1**	Allow r for radius.
	Radius of circle = $h\tan\theta$ [So $\omega^2 = \frac{6g}{7h}\cos\theta$]	**B1**	
	In second scenario, $\frac{9}{4}\omega^2 = \frac{6g}{7h}\cos\alpha$	**M1**	Second scenario, equivalent result .
	Equate, $\frac{6g}{7h}\cos\theta = \frac{4}{9}\times\frac{6g}{7h}\cos\alpha$ giving $\cos\theta = \frac{4}{9}\cos\alpha$	**A1**	Combine convincingly to obtain given result. AG
		5	

Question	Answer	Marks	Guidance
6(b)	First scenario: $N + T\cos\theta = mg$		
	Second scenario, $\frac{1}{2}N + T\cos\alpha = mg$	**B1**	Both.
	Equate: $mg - \frac{6}{7}mg\cos\theta = 2mg - \frac{12}{7}mg\cos\alpha$	**M1**	$12\cos\alpha - 6\cos\theta = 7$
	$\cos\alpha = \frac{3}{4}$	**A1**	
	$N = \frac{5}{7}mg$	**A1**	
		4	

Question	Answer	Marks	Guidance
7(a)	Let v, w be speeds of A and B along line of centres after collision $mv + \frac{1}{2}mw = mu\cos\alpha - \frac{1}{2}m.2u\cos\beta$	**M1**	Momentum: masses correct, opposite signs on RHS.
	$w - v = e(2u\cos\beta + u\cos\alpha)$	**M1**	NEL: LHS signs must be consistent with momentum equation, same sign for both terms on RHS.
	$\alpha + \beta = 90°$, so $\cos\beta = \sin\alpha$ Use this fact and solve to find w	**M1**	Solve to find an expression of the correct form.
	$w = \frac{2}{3}u\left(\frac{1}{4}\sin\alpha + \frac{13}{8}\cos\alpha\right)$	**A1**	
		4	

Question	Answer	Marks	Guidance
7(b)	Perpendicular to line of centres, speed of B is $2u\sin\beta = 2u\cos\alpha$	**B1**	
	After, velocity of B makes angle α with line of centres, so $\tan\alpha = \dfrac{2u\cos\alpha}{w}$	**B1**	
	$\dfrac{\sin\alpha}{\cos\alpha} = \dfrac{2u\cos\alpha}{\frac{2}{3}u\left(\frac{1}{4}\sin\alpha + \frac{13}{8}\cos\alpha\right)}$ giving	**M1***	Obtain homogeneous equation in cos and sin or an equation in tan
	$3(\cos\alpha)^2 = \frac{1}{4}(\sin\alpha)^2 + \frac{13}{8}\sin\alpha\cos\alpha$ $2(\tan\alpha)^2 + 13\tan\alpha - 24 = 0$, $(2\tan\alpha - 3)(\tan\alpha + 8) = 0$	**DM1**	Obtain quadratic and solve to find values of $\tan\alpha$
	$\tan\alpha = \dfrac{3}{2}$	**A1**	
		5	

Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER | | | | | |

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5661734196

FURTHER MATHEMATICS **9231/32**

Paper 3 Further Mechanics **May/June 2022**

1 hour 30 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.
- Where a numerical value for the acceleration due to gravity (g) is needed, use $10\,\text{m}\,\text{s}^{-2}$.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Any blank pages are indicated.

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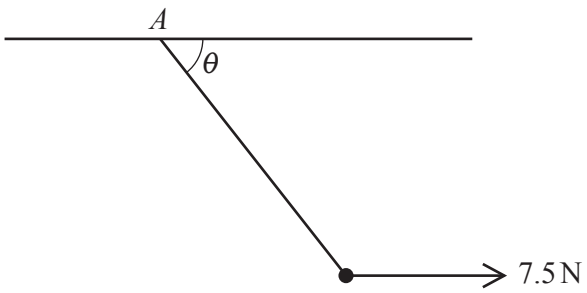

A particle of weight 10 N is attached to one end of a light elastic string. The other end of the string is attached to a fixed point A on a horizontal ceiling. A horizontal force of 7.5 N acts on the particle. In the equilibrium position, the string makes an angle θ with the ceiling (see diagram). The string has natural length 0.8 m and modulus of elasticity 50 N.

(a) Find the tension in the string. [2]

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(b) Find the vertical distance between the particle and the ceiling. [3]

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[Turn over

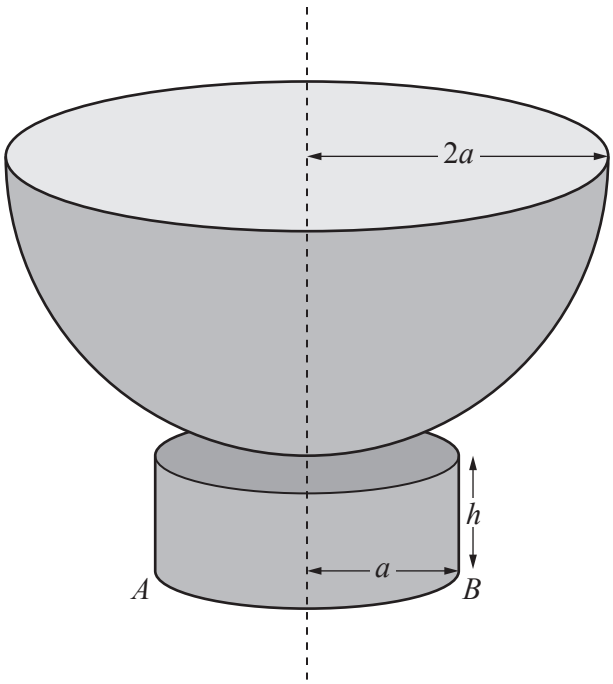
2 One end of a light inextensible string of length a is attached to a fixed point O. A particle of mass m is attached to the other end of the string. The particle is held at the point A with the string taut. The angle between OA and the downward vertical is equal to α, where $\cos\alpha = \frac{4}{5}$. The particle is projected from A, perpendicular to the string in an upwards direction, with a speed $\sqrt{3ga}$. It then moves along a circular path in a vertical plane. The string first goes slack when it makes an angle θ with the upward vertical through O.

Find the value of $\cos\theta$. [5]

3 A particle P is moving in a horizontal straight line. Initially P is at the point O on the line and is moving with velocity $25\,\text{m s}^{-1}$. At time t s after passing through O, the acceleration of P is $\frac{4000}{(5t+4)^3}\,\text{m s}^{-2}$ in the direction PO. The displacement of P from O at time t is x m.

Find an expression for x in terms of t. [5]

4



An object is composed of a hemispherical shell of radius $2a$ attached to a closed hollow circular cylinder of height h and base radius a. The hemispherical shell and the hollow cylinder are made of the same uniform material. The axes of symmetry of the shell and the cylinder coincide. AB is a diameter of the lower end of the cylinder (see diagram).

(a) Find, in terms of a and h, an expression for the distance of the centre of mass of the object from AB. [4]

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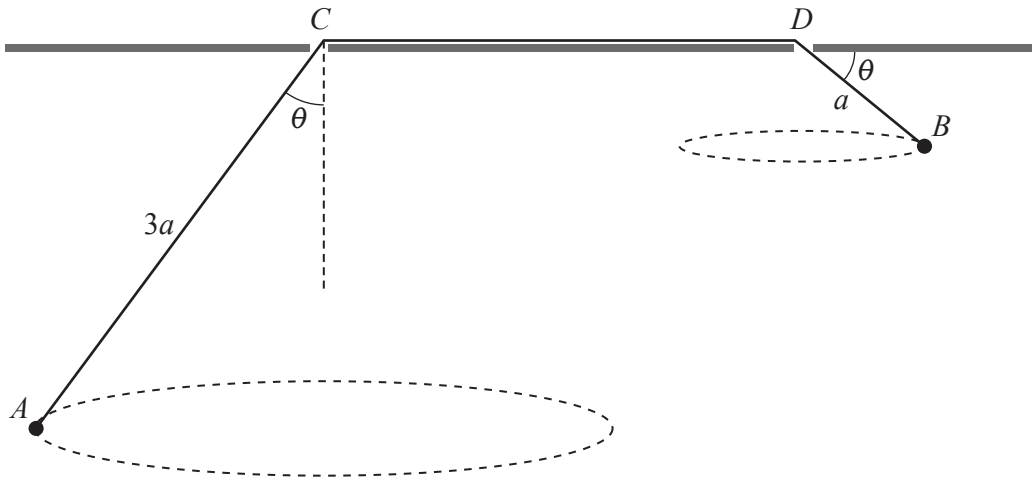
The object is placed on a rough plane which is inclined to the horizontal at an angle θ, where $\tan\theta = \frac{2}{3}$. The object is in equilibrium with AB in contact with the plane and lying along a line of greatest slope of the plane.

(b) Find the set of possible values of h, in terms of a. [4]

5



A light inextensible string AB passes through two small holes C and D in a smooth horizontal table where $AC = 3a$ and $DB = a$. A particle of mass m is attached at the end A and moves in a horizontal circle with angular velocity ω. A particle of mass $\frac{3}{4}m$ is attached to the end B and moves in a horizontal circle with angular velocity $k\omega$. AC makes an angle θ with the downward vertical and DB makes an angle θ with the horizontal (see diagram).

Find the value of k. [7]

6 Two uniform smooth spheres A and B of equal radii have masses m and km respectively. The two spheres are on a horizontal surface. Sphere A is travelling with speed u towards sphere B which is at rest. The spheres collide. Immediately before the collision, the direction of motion of A makes an angle α with the line of centres. The coefficient of restitution between the spheres is $\frac{1}{2}$.

(a) Show that the speed of B after the collision is $\frac{3u\cos\alpha}{2(1+k)}$ and find also an expression for the speed of A along the line of centres after the collision, in terms of k, u and α. [4]

After the collision, the kinetic energy of A is equal to the kinetic energy of B.

(b) Given that $\tan\alpha = \frac{2}{3}$, find the possible values of k. [5]

7 Particles P and Q are projected in the same vertical plane from a point O at the top of a cliff. The height of the cliff exceeds 50 m. Both particles move freely under gravity. Particle P is projected with speed $\frac{35}{2}\,\text{m s}^{-1}$ at an angle α above the horizontal, where $\tan\alpha = \frac{4}{3}$. Particle Q is projected with speed $u\,\text{m s}^{-1}$ at an angle β above the horizontal, where $\tan\beta = \frac{1}{2}$. Particle Q is projected one second after the projection of particle P. The particles collide T s after the projection of particle Q.

(a) Write down expressions, in terms of T, for the horizontal displacements of P and Q from O when they collide and hence show that $4uT = 21\sqrt{5}(T+1)$. [4]

(b) Find the value of T. [4]

(c) Find the horizontal and vertical displacements of the particles from O when they collide. [3]

Additional page

If you use the following page to complete the answer to any question, the question number must be clearly shown.

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FURTHER MATHEMATICS **9231/32**

Paper 3 Further Mechanics **May/June 2022**

MARK SCHEME

Maximum Mark: 50

Published

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Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2022 series for most Cambridge IGCSE, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

This document consists of **14** printed pages.

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WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1(a)	$T\cos\theta = 7.5, \quad T\sin\theta = 10$	**B1**	
	$T = \left(7.5^2 + 10^2\right)^{\frac{1}{2}} = 12.5\text{ N}$	**B1**	
		2	
1(b)	Hooke's law: $T = \frac{50x}{0.8}$, $x = 0.2$	**B1**	
	$(x + 0.8)\sin\theta = 1 \times \frac{10}{12.5}$	**M1**	
	Vertical distance $= \frac{4}{5} = 0.8$	**A1**	
		3	

Question	Answer	Marks	Guidance
2	$-\frac{1}{2}mv^2 + \frac{1}{2}m \times 3ga = mga(\cos\theta + \cos\alpha)$	**M1 A1**	Energy equation.
	$(T +)mg\cos\theta = \frac{m}{a}v^2$	**B1**	N2L
	$ag\cos\theta = 3ga - 2ga\cos\theta - 2ga \times \frac{4}{5}$	**M1**	Combine to find $\cos\theta$.
	$\cos\theta = \frac{7}{15}$	**A1**	
		5	

Question	Answer	Marks	Guidance
3	$\frac{dv}{dt} = -\frac{4000}{(5t+4)^3}$; $v = \frac{400}{(5t+4)^2}(+A)$	**M1 A1**	Integrate. Constant of integration needed for A1.
	$t = 0, \quad v = 25 \quad A = 25 - 25 = 0$	**M1**	Find constant.
	$v = \frac{dx}{dt}$: $x = -80(5t+4)^{-1} + B$ $x = 0, \ t = 0 \quad B = 20$	**M1**	Integrate and find constant.
	$x = \frac{-80}{5t+4} + 20 \quad \left(= \frac{100t}{5t+4}\right)$	**A1**	
		5	

Question	Answer	Marks	Guidance
4(a)	<table><tr><th></th><th>Area</th><th>Centre of mass from AB</th></tr><tr><td>Cylinder</td><td>$2\pi ah + 2\pi a^2$</td><td>$\frac{1}{2}h$</td></tr><tr><td>Shell</td><td>$2\pi(2a)^2$</td><td>$h + a$</td></tr></table>	**M1**	Moments equation, condone missing ends of cylinder. One expression on the RHS correct.
	Moments about AB $\bar{x}\left(2\pi ah + 2\pi a^2 + 2\pi(2a)^2\right) = 2\pi(2a)^2 \times (h+a) + \left(2\pi ah + 2\pi a^2\right)\left(\frac{1}{2}h\right)$	**A1 A1**	One correct expression on RHS correct scores A1.
	$(2h + 10a)\bar{x} = h^2 + ah + 8ah + 8a^2$ $\bar{x} = \frac{h^2 + 9ah + 8a^2}{2(h+5a)}$	**A1**	
		4	
4(b)	$\tan\theta \leqslant \frac{a}{\bar{x}}$	**B1**	
	$\bar{x} = \frac{h^2 + 9ah + 8a^2}{2(h+5a)} \leqslant \frac{3}{2}a$ $h^2 + 6ah - 7a^2 \leqslant 0$	**M1**	Form inequality and rearrange to quadratic, condone equation.
	$(h-a)(h+7a) \leqslant 0$	**M1**	Attempt to solve, condone equation.
	$(-7a \leqslant)\, h \leqslant a$	**A1**	
		4	

Question	Answer	Marks	Guidance
5	For A: $T\sin\theta = mr\omega^2$	**M1**	N2L horizontal.
	$r = 3a\sin\theta$	**B1**	Correct expression for radius.
	$T = m \times 3a\omega^2$	**A1**	
	Similarly, for B: $T\cos\theta = \frac{3}{4}m \times r \times k^2\omega^2$	**M1**	N2L horizontal
	$T = \frac{3}{4}mak^2\omega^2$	**A1**	
	$m \times 3a\omega^2 = \frac{3}{4}mak^2\omega^2$	**M1**	Equate expressions for T.
	$k^2 = 4, \quad k = 2$	**A1**	

Question	Answer	Marks	Guidance
5	**Alternative method for question 5**		
	For A: $T\cos\theta = mg, \quad T\sin\theta = mr\omega^2$	**M1**	N2L horizontal and vertical.
	$r = 3a\sin\theta$	**B1**	Correct expression for radius.
	$T = m\times 3a\omega^2 = \frac{5}{4}mg, \quad \omega^2 = \frac{5}{12}\times\frac{g}{a}$	**A1**	Combine to obtain expression for ω^2.
	Similarly, for B: $T\cos\theta = \frac{3}{4}m\times r\times k^2\omega^2$	**M1**	N2L horizontal.
	$T = \frac{3}{4}mak^2\omega^2$	**A1**	
	$\frac{5}{4}mg = \frac{3}{4}mak^2\times\frac{5g}{12a}$	**M1**	Substitute for T and ω.
	$k^2 = 4, \quad k = 2$	**A1**	
		7	

Question	Answer	Marks	Guidance
6(a)	Let v and w be speeds after collision: $mv + kmw = mu\cos\alpha$	**M1**	Momentum along line of centres.
	$w - v = \frac{1}{2}u\cos\alpha$	**M1**	NEL consistent signs.
	Add to give $\frac{3u\cos\alpha}{2(1+k)}$	**A1**	AG Convincing working.
	Substitute back or re-solve: $v = \left\|\frac{(2-k)u\cos\alpha}{2(1+k)}\right\|$	**A1**	Accept without modulus sign.
		4	

Question	Answer	Marks	Guidance
6(b)	$\sqrt{(u\sin\alpha)^2+\left(\frac{(2-k)u\cos\alpha}{2(1+k)}\right)^2}$	**B1**	For speed of A (SOI).
	Equal KE after collision: $\frac{1}{2}km\left(\frac{3u\cos\alpha}{2(1+k)}\right)^2=\frac{1}{2}m\left((u\sin\alpha)^2+\left(\frac{(2-k)u\cos\alpha}{2(1+k)}\right)^2\right)$ $\left[9k(\cos\alpha)^2=4(1+k)^2(\sin\alpha)^2+(2-k)^2(\cos\alpha)^2\right]$	**M1**	Equate KEs.
	Use $\tan\alpha=\frac{2}{3}$: $16(1+2k+k^2)+9(4-4k+k^2)=81k$	**M1**	
	$25k^2-85k+52=0$ leading to $(5k-4)(5k-13)=0$	**M1**	Obtain quadratic and attempt to solve.
	$k=\frac{4}{5}$ or $\frac{13}{5}$	**A1**	
		5	

Question	Answer	Marks	Guidance
7(a)	For Q: $x = u\cos\beta T$	**B1**	
	For P: $x = \frac{35}{2}\cos\alpha(T+1)$	**B1**	
	Collision, so $\frac{35}{2}\cos\alpha(T+1) = u\cos\beta T$	**M1**	Equate and attempt to rearrange.
	$\frac{35}{2}\times\frac{3}{5}(T+1) = u\times\frac{2}{\sqrt{5}}T$ $4uT = 21\sqrt{5}(T+1)$	**A1**	AG Shown convincingly.
		4	
7(b)	Vertical motion to collision: For Q: $y = u\sin\beta T - \frac{1}{2}gT^2$ For P: $y = \frac{35}{2}\sin\alpha(T+1) - \frac{1}{2}g(T+1)^2$	**M1 A1**	M1 for both expressions, one correct.
	Equate: $u\times\frac{1}{\sqrt{5}}T - \frac{1}{2}gT^2 = \frac{35}{2}\times\frac{4}{5}(T+1) - \frac{1}{2}g(T+1)^2$ $14(T+1) - \frac{1}{2}g\left(T^2+2T+1-T^2\right) = \frac{21}{4}(T+1)$	**M1**	Equate and attempt to solve
	$16T+36 = 21T+21, \quad 15 = 5T$ $T = 3$	**A1**	
		4	

Question	Answer	Marks	Guidance
7(c)	$x = 42$	**B1**	
	$\lvert y \rvert = 24$	**M1**	
	$y = -24$ (or 24 m below O)	**A1**	Correct sign or in words.
		3	

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FURTHER MATHEMATICS **9231/33**

Paper 3 Further Mechanics **May/June 2022**

1 hour 30 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.
- Where a numerical value for the acceleration due to gravity (g) is needed, use $10\,\text{m}\,\text{s}^{-2}$.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Any blank pages are indicated.

1 A uniform lamina $OABC$ is a trapezium whose vertices can be represented by coordinates in the x-y plane. The coordinates of the vertices are $O(0,0)$, $A(15,0)$, $B(9,4)$ and $C(3,4)$.

Find the x-coordinate of the centre of mass of the lamina. [4]

2 A particle P of mass m is attached to one end of a light elastic string of natural length a and modulus of elasticity $\frac{4}{3}mg$. The other end of the string is attached to a fixed point O on a rough horizontal surface. The particle is at rest on the surface with the string at its natural length. The coefficient of friction between P and the surface is $\frac{1}{3}$. The particle is projected along the surface in the direction OP with a speed of $\frac{1}{2}\sqrt{ga}$.

Find the greatest extension of the string during the subsequent motion. [5]

3 A particle P is projected with speed $25\,\text{m s}^{-1}$ at an angle θ above the horizontal from a point O on a horizontal plane and moves freely under gravity. After 2 s the speed of P is $15\,\text{m s}^{-1}$.

(a) Find the value of $\sin\theta$. [5]

(b) Find the range of the flight. [3]

4 One end of a light inextensible string of length a is attached to a fixed point O. A particle of mass m is attached to the other end of the string and is held with the string taut at the point A. At A the string makes an angle θ with the upward vertical through O. The particle is projected perpendicular to the string in a downward direction from A with a speed u. It moves along a circular path in the vertical plane.

When the string makes an angle α with the downward vertical through O, the speed of the particle is $2u$ and the magnitude of the tension in the string is 10 times its magnitude at A.

It is given that $u = \sqrt{\frac{2}{3}ga}$.

(a) Find, in terms of m and g, the magnitude of the tension in the string at A. [6]

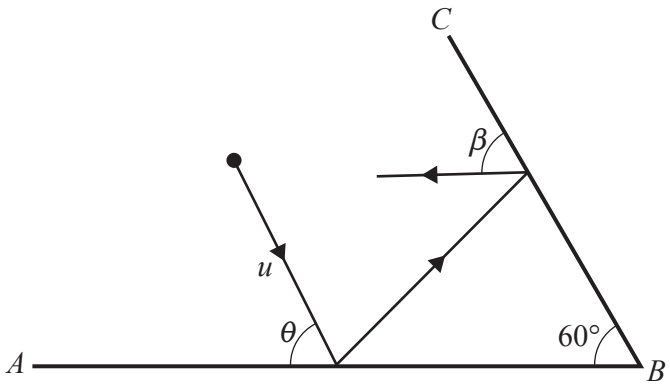
(b) Find the value of $\cos\alpha$. [2]

5 A particle P of mass 4 kg is moving in a horizontal straight line. At time t s the velocity of P is v m s^{-1} and the displacement of P from a fixed point O on the line is x m. The only force acting on P is a resistive force of magnitude $(4e^{-x} + 12)e^{-x}$ N. When $t = 0$, $x = 0$ and $v = 4$.

(a) Show by integration that $v = \dfrac{1 + 3e^{x}}{e^{x}}$. [4]

(b) Find an expression for x in terms of t. [4]

6



AB and BC are two fixed smooth vertical barriers on a smooth horizontal surface, with angle $ABC = 60°$. A particle of mass m is moving with speed u on the surface. The particle strikes AB at an angle θ with AB. It then strikes BC and rebounds at an angle β with BC (see diagram). The coefficient of restitution between the particle and each barrier is e and $\tan\theta = 2$.

The kinetic energy of the particle after the first collision is 40% of its kinetic energy before the first collision.

(a) Find the value of e. [4]

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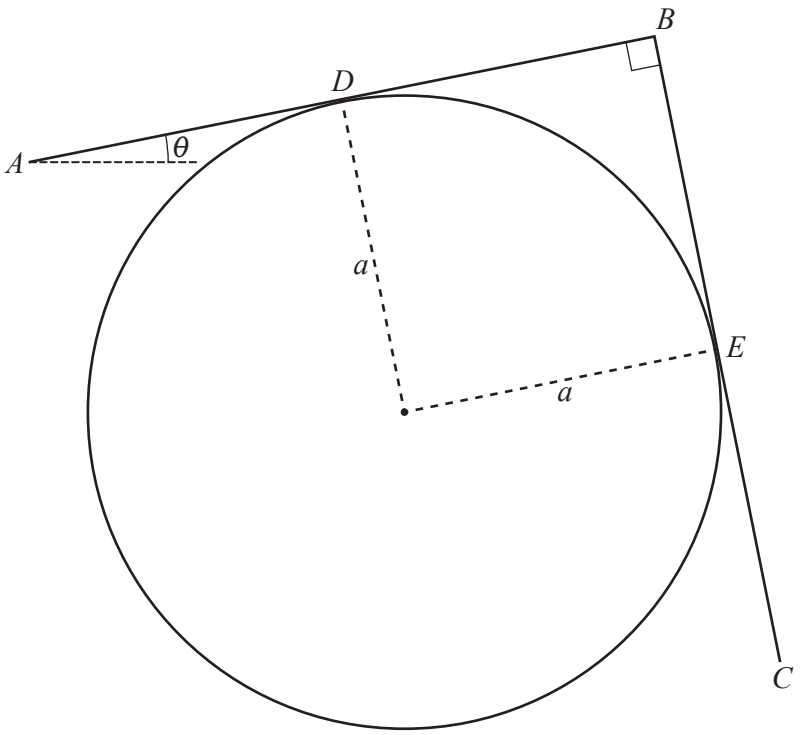
(b) Find the size of angle β. [4]

7



A uniform cylinder with a rough surface and of radius a is fixed with its axis horizontal. Two identical uniform rods AB and BC, each of weight W and length $2a$, are rigidly joined at B with AB perpendicular to BC. The rods rest on the cylinder in a vertical plane perpendicular to the axis of the cylinder with AB at an angle θ to the horizontal. D and E are the midpoints of AB and BC respectively and also the points of contact of the rods with the cylinder (see diagram). The rods are about to slip in a clockwise direction. The coefficient of friction between each rod and the cylinder is μ.

The normal reaction between AB and the cylinder is R and the normal reaction between BC and the cylinder is N.

(a) Find the ratio $R : N$ in terms of μ. [6]

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(b) Given that $\mu = \frac{1}{3}$, find the value of $\tan\theta$. [3]

Additional page

If you use the following page to complete the answer to any question, the question number must be clearly shown.

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Cambridge Assessment International Education

Cambridge International AS & A Level

FURTHER MATHEMATICS **9231/33**

Paper 3 Further Mechanics **May/June 2022**

MARK SCHEME

Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2022 series for most Cambridge IGCSE, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

This document consists of **13** printed pages.

PUBLISHED

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:
Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:
Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:
Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:
Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

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Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	<table><tr><th></th><th>Area</th><th>Distance from Oy</th></tr><tr><td>Triangle OCD</td><td>6</td><td>2</td></tr><tr><td>Rectangle $DEBC$</td><td>24</td><td>6</td></tr><tr><td>Triangle BAE</td><td>12</td><td>11</td></tr><tr><td>Trapezium $OCBA$</td><td>42</td><td>$\bar{x}$</td></tr></table> where D is point with coordinates (3, 0) and E is a point with coordinates (9, 0).	**M1**	Attempt at moments equation with all necessary terms. Other options possible for RHS of moments equation, for example: (1) OAC: 30×6 and ABC: 12×9 (2) OBC: 12×4 and OAB: 30×8 (3) Subtraction: $60\times 7.5-6\times 1-12\times 13$
	Parts that would give correct total area 42	**B1**	
	Moments about Oy $42\bar{x}=6\times 2+24\times 6+12\times 11$ (=288)	**A1**	Correct equation.
	$\bar{x}=\dfrac{288}{42}=6.86$	**A1**	
		4	

Question	Answer	Marks	Guidance
2	$\frac{1}{2}.\frac{\frac{4}{3}mg}{a}x^2$	**B1**	EPE term correct
	$\frac{1}{3}mgx$	**B1**	Work term correct
	Loss in KE = gain in EPE + work done against friction $\frac{1}{2}mv^2 = \frac{1}{2}\times\frac{\frac{4}{3}mg}{a}x^2 + \frac{1}{3}mgx$	**M1**	Energy equation with 3 terms, allow sign error.
	$\frac{1}{2}\times\frac{1}{4}ga = \frac{\frac{2}{3}g}{a}x^2 + \frac{1}{3}gx$ $16x^2 + 8ax - 3a^2 = 0$ $(4x - a)(4x + 3a) = 0$	**M1**	Obtain and attempt to solve a 3-term quadratic equation.
	$x = \frac{1}{4}a$	**A1**	
		5	

Question	Answer	Marks	Guidance
3(a)	Components of velocity : $\rightarrow 25\cos\theta$ $\uparrow 25\sin\theta - 2g$	**B1**	
	Speed = $\sqrt{(25\cos\theta)^2 + (25\sin\theta - 2g)^2}$	**M1 A1**	Expression for speed or square of speed.
	$(25\cos\theta)^2 + (25\sin\theta - 2g)^2 = 15^2$ $625 - 100g\sin\theta + 4g^2 = 225$	**M1**	Attempt to solve and find value for $\sin\theta$
	$\sin\theta = \frac{800}{1000} = \frac{4}{5}$	**A1**	
		5	
3(b)	Time of flight $= \left(\frac{2\times 25\sin\theta}{g}\right) = 4$ (s)	**B1**	
	Range = $\frac{2\times 25\sin\theta}{g}\times 25\cos\theta$	**M1**	Any equivalent method.
	Range = 60 (m)	**A1**	CWO
	Alternative method for question 3(b)		
	$y = \frac{4}{3}x - \frac{1}{45}x^2$	**B1**	Equation of trajectory..
	Substitute $y = 0$ and solve	**M1**	
	60 (m)	**A1**	
		3	

Question	Answer	Marks	Guidance
4(a)	$\frac{1}{2}mv^2 - \frac{1}{2}mu^2 = mga(\cos\theta + \cos\alpha)$	**M1**	Energy equation with all necessary terms, GPE terms must be resolved, allow sin/cos mix, allow sign error.
	$\frac{1}{2}m(2u)^2 - \frac{1}{2}mu^2 = mga(\cos\theta + \cos\alpha)$	**A1**	$2u$ may be substituted later. Implied by $\frac{3}{2}\times\frac{2}{3}ag = ga(\cos\theta + \cos\alpha)$
	At A, $T + mg\cos\theta = \frac{m}{a}u^2$	**B1**	N2L
	Also, $10T - mg\cos\alpha = \frac{m}{a}4u^2$	**B1**	N2L and use of tension ($10T$).
	Use all three (two N2L and energy) equations to find T in terms of m and g only.	**M1**	Might see $9T - mg(\cos\theta + \cos\alpha) = \frac{3m}{a}\times\frac{2}{3}ga$ $(\cos\theta + \cos\alpha) = 1$ $(10\cos\theta + \cos\alpha) = 4$
	$T = \frac{1}{3}mg$	**A1**	
		6	
4(b)	Substitute back, $10\times\frac{1}{3}mg - mg\cos\alpha = \frac{4m}{a}\times\frac{2}{3}ga$	**M1**	Any appropriate method to obtain $\cos\alpha$.
	$\cos\alpha = \frac{2}{3}$	**A1**	
		2	

Question	Answer	Marks	Guidance
5(a)	$4v\frac{dv}{dx} = -\left(4\mathrm{e}^{-x} + 12\right)\mathrm{e}^{-x}$	**B1**	
	$\frac{1}{2}v^2 = \frac{1}{2}\mathrm{e}^{-2x} + 3\mathrm{e}^{-x}\left(+A\right)$	**M1**	Expression of the correct form.
	$v = 4, \quad x = 0, \quad A = \frac{9}{2}$	**A1**	
	$v^2 = \mathrm{e}^{-2x} + 6\mathrm{e}^{-x} + 9 = \left(3 + \mathrm{e}^{-x}\right)^2$ $v = 3 + \mathrm{e}^{-x} = \frac{1 + 3\mathrm{e}^{x}}{\mathrm{e}^{x}}$	**A1**	AG Must see the factorisation. Condone lack of justification for taking positive square root.
		4	
5(b)	$\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{1 + 3\mathrm{e}^{x}}{\mathrm{e}^{x}}$ so $\int \frac{\mathrm{e}^{x}}{3\mathrm{e}^{x} + 1}dx = \int 1\ \mathrm{d}t$ $\frac{1}{3}\ln\left(3\mathrm{e}^{x} + 1\right) = t\left(+B\right)$	**M1* A1**	Integration to obtain ln term Correct answer with constant of integration
	$t = 0, \quad x = 0, \quad B = \frac{1}{3}\ln 4$ $3t = \ln\frac{3\mathrm{e}^{x} + 1}{4}$	**DM1**	Find the constant and substitute into their general solution.
	$x = \ln\left(\frac{4}{3}\mathrm{e}^{3t} - \frac{1}{3}\right)$	**A1**	OE
		4	

Question	Answer	Marks	Guidance
6(a)	Let v be speed of rebound from 1st collision: Energy loss: $\frac{1}{2}mv^2 = \frac{2}{5}\times\frac{1}{2}mu^2$, $v^2 = \frac{2}{5}u^2$	**B1**	Energy loss.
	$v\cos\alpha = u\cos\theta$ $v\sin\alpha = eu\sin\theta$	**B1**	Both.
	Combine to form equation in e only $\frac{2}{5} = \frac{1}{5} + e^2\times\frac{4}{5}$	**M1**	$v^2 = (u\cos\theta)^2 + (eu\sin\theta)^2$
	$e = \frac{1}{2}$	**A1**	
		4	
6(b)	$\tan\alpha = e\tan\theta$, so $\tan\alpha = 1$, $\alpha = 45°$	**B1**	
	For 2nd collision $w\cos\beta = v\cos(180-60-\alpha)$ $w\sin\beta = ev\sin(180-60-\alpha)$	**M1**	Both. May be implied by the A1.
	$\tan\beta = e\tan(120 - \textit{their}\,\alpha)$	**M1**	Divide to find β.
	$\beta = 61.8°$	**A1**	
		4	

Question	Answer	Marks	Guidance
7(a)	Frictional force = $\mu\times$ normal reaction at D and E	**B1**	$F_{AB}=\mu R,\ F_{BC}=\mu N$
	Moments about B, $Na-Ra=Wa(\sin\theta-\cos\theta)$ Moments about centre, $F_{AB}a+F_{BC}a=Wa(\cos\theta-\sin\theta)$ Moments about D, $F_{BC}a+Na=Wa(\cos\theta+\sin\theta)$ Moments about E, $Ra-F_{AB}a=Wa(\cos\theta+\sin\theta)$	**B1**	One moments equation about any point involving all relevant forces, resolved if necessary (AEF).
	Parallel to AB, $N-F_{AB}=W\sin\theta+W\sin\theta$ Perpendicular to AB, $F_{BC}+R=W\cos\theta+W\cos\theta$	**B1**	Two resolutions: all relevant terms, different frictional forces [Vertical: $R\cos\theta+F_{BC}\cos\theta+N\sin\theta=F_{AB}\sin\theta+W+W$ Horizontal: $F_{BC}\sin\theta+F_{AB}\cos\theta+R\sin\theta=N\cos\theta$] Alternative approach using two moments equations can earn the B1B1
	$N-R=\frac{1}{2}\left((1-\mu)N-(1+\mu)R\right)$	**M1**	Combine appropriate equations.
	$N\left(1-\frac{1}{2}(1-\mu)\right)=R\left(1-\frac{1}{2}(1+\mu)\right)$ $N\left(\frac{1}{2}+\frac{1}{2}\mu\right)=R\left(\frac{1}{2}-\frac{1}{2}\mu\right)$	**M1**	Collect terms to obtain ratio/fraction in terms of μ only (CWO), any equivalent simplified form.
	$R:N=1+\mu:1-\mu$	**A1**	
		6	

Question	Answer	Marks	Guidance
7(b)	Divide resolution equations: $\tan\theta = \dfrac{N-\mu R}{\mu N + R}$	**M1**	Must include μ terms.
	Use $R = 2N$: $\tan\theta = \dfrac{\frac{1}{3}N}{\frac{7}{3}N}$	**M1**	FT their answer to part **(a)**.
	$\tan\theta = \dfrac{1}{7}$	**A1**	
		3	