Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME									
CENTRE NUMBER						CANDIDATE NUMBER			

FURTHER MATHEMATICS **9231/31**

Paper 3 Further Mechanics **October/November 2021**

1 hour 30 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.
- Where a numerical value for the acceleration due to gravity (g) is needed, use $10\,\text{m}\,\text{s}^{-2}$.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Any blank pages are indicated.

BLANK PAGE

1 One end of a light elastic string, of natural length a and modulus of elasticity $3mg$, is attached to a fixed point O on a smooth horizontal plane. A particle P of mass m is attached to the other end of the string and moves in a horizontal circle with centre O. The speed of P is $\sqrt{\frac{4}{3}ga}$.

Find the extension of the string. [4]

2 A particle P of mass m kg moves along a horizontal straight line with acceleration $a\,\text{m s}^{-2}$ given by

$$a = \frac{v(1-2t^2)}{t},$$

where $v\,\text{m s}^{-1}$ is the velocity of P at time t s.

(a) Find an expression for v in terms of t and an arbitrary constant. [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(b) Given that $a = 5$ when $t = 1$, find an expression, in terms of m and t, for the horizontal force acting on P at time t. [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

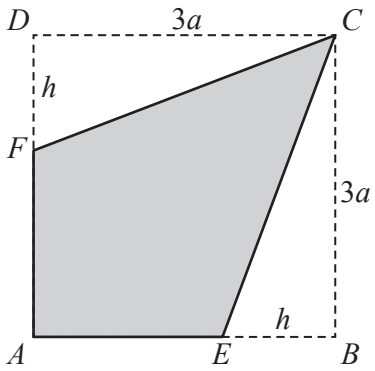
.....

.....

3 A light elastic string has natural length a and modulus of elasticity $12mg$. One end of the string is attached to a fixed point O. The other end of the string is attached to a particle of mass m. The particle hangs in equilibrium vertically below O. The particle is pulled vertically down and released from rest with the extension of the string equal to e, where $e > \frac{1}{3}a$. In the subsequent motion the particle has speed $\sqrt{2ga}$ when it has ascended a distance $\frac{1}{3}a$.

Find e in terms of a. [6]

4



A uniform lamina $AECF$ is formed by removing two identical triangles BCE and CDF from a square lamina $ABCD$. The square has side $3a$ and $EB = DF = h$ (see diagram).

(a) Find the distance of the centre of mass of the lamina $AECF$ from AD and from AB, giving your answers in terms of a and h. [5]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

The lamina $AECF$ is placed vertically on its edge AE on a horizontal plane.

(b) Find, in terms of a, the set of values of h for which the lamina remains in equilibrium. [3]

[Turn over

5 A particle P is projected from a point O on a horizontal plane and moves freely under gravity. Its initial speed is $u\,\text{m s}^{-1}$ and its angle of projection is $\sin^{-1}(\frac{4}{5})$ above the horizontal. At time 8 s after projection, P is at the point A. At time 32 s after projection, P is at the point B. The direction of motion of P at B is perpendicular to its direction of motion at A.

Find the value of u. [7]

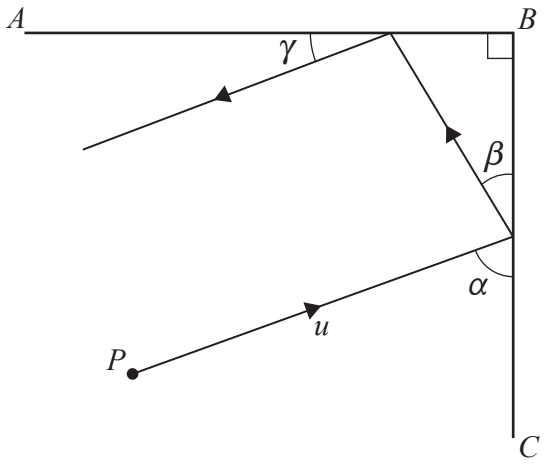
6 A particle P, of mass m, is attached to one end of a light inextensible string of length a. The other end of the string is attached to a fixed point O. The particle P moves in complete vertical circles about O with the string taut. The points A and B are on the path of P with AB a diameter of the circle. OA makes an angle θ with the downward vertical through O and OB makes an angle θ with the upward vertical through O. The speed of P when it is at A is $\sqrt{5ag}$.

The ratio of the tension in the string when P is at A to the tension in the string when P is at B is 9 : 5.

(a) Find the value of $\cos\theta$. [6]

(b) Find, in terms of a and g, the greatest speed of P during its motion. [2]

7



The smooth vertical walls AB and CB are at right angles to each other. A particle P is moving with speed u on a smooth horizontal floor and strikes the wall CB at an angle α. It rebounds at an angle β to the wall CB. The particle then strikes the wall AB and rebounds at an angle γ to that wall (see diagram). The coefficient of restitution between each wall and P is e.

(a) Show that $\tan\beta = e\tan\alpha$. [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(b) Express γ in terms of α and explain what this result means about the final direction of motion of P. [4]

.....

.....

.....

.....

.....

.....

.....

As a result of the two impacts the particle loses $\frac{8}{9}$ of its initial kinetic energy.

(c) Given that $\alpha + \beta = 90°$, find the value of e and the value of $\tan\alpha$. [4]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

BLANK PAGE

BLANK PAGE

Cambridge Assessment International Education

Cambridge International AS & A Level

FURTHER MATHEMATICS **9231/31**

Paper 3 Further Mechanics **October/November 2021**

MARK SCHEME

Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2021 series for most Cambridge IGCSE™, Cambridge International A and AS Level components and some Cambridge O Level components.

[Turn over

PUBLISHED

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:
Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:
Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:
Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:
Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

PUBLISHED

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	$T = \dfrac{3mgx}{a}$	**B1**	Their tensions equated to obtain a quadratic equation, CAO.
	$T = \dfrac{4mga}{3(a+x)}$	**B1**	
	$9x^2 + 9ax - 4a^2 = 0$ leading to $(3x-a)(3x+4a)=0$	**M1**	
	$x = \dfrac{1}{3}a$	**A1**	
		4	
2(a)	Separate variables and integrate: $\dfrac{dv}{v} = \left(\dfrac{1-2t^2}{t}\right)dt$ so $\ln\lvert v\rvert = \ln t - t^2 + c$	**M1 A1**	Must include logs. Condone missing modulus.
	$\lvert v\rvert = Ate^{-t^2}$, $-v = Ate^{-t^2}$, $v = -Ate^{-t^2}$	**A1**	CAO.
		3	

Question	Answer	Marks	Guidance
2(b)	$a = \dfrac{-Ate^{-t^2}\left(1-2t^2\right)}{t} = -Ae^{-t^2}\left(1-2t^2\right)$	**M1**	Substituting their answer to part **(a)** into given formula
	$t=1,\ a=5 \quad (A=5e)$	**M1**	Use initial condition.
	Force = $5me^{1-t^2}\left(2t^2-1\right)$	**A1**	Use N2L, correct work only.
	Alternative method for question 2(b)		
	$a = \dfrac{v\left(1-2t^2\right)}{t}$ substitute $t=1,\ a=5$ so $v=-5$	**M1**	Use initial condition. Use N2L, correct work only.
	Substituting in their answer to part **(a)** so $(A=5e)$	**M1**	
	Force = $5me^{1-t^2}\left(2t^2-1\right)$	**A1**	
		3	

Question	Answer	Marks	Guidance
3	Loss in EPE = $\frac{1}{2}\times\frac{12mge^2}{a}-\frac{1}{2}\times\frac{12mg}{a}\times\left(e-\frac{a}{3}\right)^2$ $\left(=\frac{2mg}{3}(6e-a)\right)$	**B1**	Either term correct.
	Gain in KE = $\frac{1}{2}mv^2$ and Gain in GPE = $\frac{mga}{3}$	**B1**	
	Gain in KE + Gain in GPE = Loss in EPE	**M1**	KE, GPE and at least one EPE term.
	$\frac{1}{2}mv^2+\frac{mga}{3}=\frac{2mg}{3}(6e-a)$	**A1**	All terms correct.
	Simplify to a linear equation in *e*.	**M1**	
	$e=\frac{1}{2}a$	**A1**	
		6	

Question	Answer	Marks	Guidance
4(a)	<table><tr><th></th><th>Area</th><th>Centre of mass from AD</th></tr><tr><td>Square</td><td>$9a^2$</td><td>$\frac{3}{2}a$</td></tr><tr><td>CDF</td><td>$\frac{3}{2}ah$</td><td>a</td></tr><tr><td>BEC</td><td>$\frac{3}{2}ah$</td><td>$3a-\frac{1}{3}h$</td></tr><tr><td>Resulting $AEFC$</td><td>$9a^2-3ah$</td><td>$\bar{x}$</td></tr></table>	**M1**	Attempt at moments with three terms.
	Taking moments about AD: $\left(9a^2-3ah\right)\ \bar{x}=\left(9a^2\times\frac{3}{2}a\right)-\left(\frac{3}{2}ah\times a\right)-\left(\frac{3}{2}ah\times\left(3a-\frac{1}{3}h\right)\right)$	**A1** **A1**	Two terms correct. All correct.
	$\bar{x}=\frac{27a^2-12ah+h^2}{6(3a-h)}\ \left(=\frac{9a-h}{6}\right)$	**A1**	AEF
	$\bar{y}=\bar{x}$	**B1**	By symmetry or equal to their $\bar{x}$.
		5	
4(b)	For equilibrium, $\bar{x} \leqslant 3a-h$ $27a^2-12ah+h^2 \leqslant 6(3a-h)^2$	**B1**	Accept strict inequality.
	$27a^2-24ah+5h^2 \geqslant 0$	**M1**	Homogeneous 3-term quadratic inequality.
	$h \leqslant \frac{9}{5}a$	**A1**	CAO.
		3	

Question	Answer	Marks	Guidance
5	At A: $\uparrow u\sin\theta - 8g$ $\quad\rightarrow u\cos\theta$	**M1**	Both.
	$\tan\alpha = \dfrac{u\sin\theta - 8g}{u\cos\theta}$	**A1**	
	At B: $\uparrow u\sin\theta - 32g$ $\quad\rightarrow u\cos\theta$	**M1**	Both.
	$\tan\beta = \dfrac{u\sin\theta - 32g}{u\cos\theta}$	**A1**	
	$\dfrac{u\sin\theta - 8g}{u\cos\theta} \times \dfrac{u\sin\theta - 32g}{u\cos\theta} = -1$	**B1**	Perpendicular directions, so $\tan\alpha \times \tan\beta = -1$.
	$u^2 - 320u + 25600 = 0$	**M1**	Simplify to a quadratic in u.
	$u = 160$	**A1**	
		7	

Question	Answer	Marks	Guidance
6(a)	At A $T_A - mg\cos\theta = m\times\frac{5ag}{a}$	**B1**	N2L
	At B $T_B + mg\cos\theta = m\times\frac{v^2}{a}$	**B1**	N2L
	$\frac{1}{2}m\times 5ag - \frac{1}{2}mv^2 = mga\times 2\cos\theta$	**M1**	Energy equation with correct number of terms.
	$v^2 = 5ag - 4ga\cos\theta$	**A1**	Accept multiplied by m and/or divided by a.
	Use ratio of tensions = 9 : 5	**M1**	Use ratio and simplify to an expression in $\cos\theta$.
	$\cos\theta = \frac{2}{5}$	**A1**	CAO
		6	
6(b)	Greatest speed at lowest point $-\frac{1}{2}m\times 5ag + \frac{1}{2}mV^2 = mga\times(1-\cos\theta)$	**M1**	Energy equation including lowest point, correct number of terms.
	$V = \sqrt{\frac{31ag}{5}}$	**A1 FT**	Ft their $\cos\theta$ from part **(a)**.
		2	

Question	Answer	Marks	Guidance
7(a)	$u\cos\alpha = v\cos\beta$	**M1**	
	$eu\sin\alpha = v\sin\beta$	**M1**	
	Divide: $\tan\beta = e\tan\alpha$	**A1**	AG. Must see divide OE.
		3	
7(b)	$v\sin\beta = w\cos\gamma \quad (= eu\sin\alpha)$	**M1**	
	$ev\cos\beta = w\sin\gamma \quad (= eu\cos\alpha)$	**M1**	
	Divide: $\tan\gamma = 1/\tan\alpha$: $\gamma = 90° - \alpha$	***A1**	
	After second rebound, direction of motion is parallel to initial path.	**DB1**	
		4	
7(c)	Final KE $= \frac{1}{2}m\left((eu\sin\alpha)^2 + (eu\cos\alpha)^2\right) \quad \left(= \frac{1}{2}me^2u^2\right)$	**M1**	Energy expression in terms of u.
	So $\frac{1}{2}me^2u^2 = \frac{1}{9}\times\frac{1}{2}mu^2$ giving $e = \frac{1}{3}$	**A1**	
	Part **(a)** gives $\tan(90-\alpha) = e\tan\alpha$	**M1**	
	So $\tan\alpha = \sqrt{3}$	**A1**	
		4	

Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME	
CENTRE NUMBER	
CANDIDATE NUMBER	

FURTHER MATHEMATICS **9231/32**

Paper 3 Further Mechanics **October/November 2021**

1 hour 30 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.
- Where a numerical value for the acceleration due to gravity (g) is needed, use $10\,\text{m}\,\text{s}^{-2}$.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Any blank pages are indicated.

[Turn over

1 A particle is projected with speed u at an angle α above the horizontal from a point O on a horizontal plane. The particle moves freely under gravity.

(a) Write down the horizontal and vertical components of the velocity of the particle at time T after projection. [2]

.....

.....

.....

.....

.....

At time T after projection, the direction of motion of the particle is perpendicular to the direction of projection.

(b) Express T in terms of u, g and α. [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

(c) Deduce that $T > \frac{u}{g}$. [1]

.....

.....

.....

.....

.....

.....

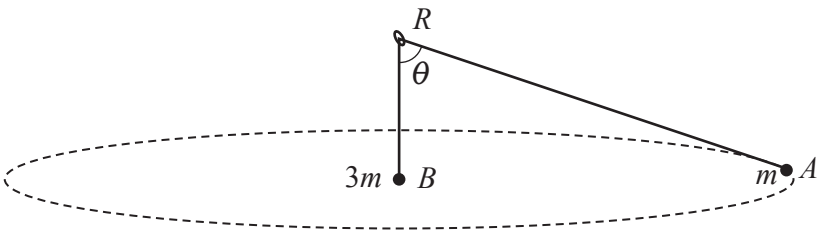
.....

2 A light spring AB has natural length a and modulus of elasticity $5mg$. The end A of the spring is attached to a fixed point on a smooth horizontal surface. A particle P of mass m is attached to the end B of the spring. The spring and particle P are at rest on the surface.

Another particle Q of mass km is moving with speed $\sqrt{4ga}$ along the horizontal surface towards P in the direction BA. The particles P and Q collide directly and coalesce. In the subsequent motion the greatest amount by which the spring is compressed is $\frac{1}{5}a$.

Find the value of k. [6]

3



Particles A and B, of masses m and $3m$ respectively, are connected by a light inextensible string of length a that passes through a fixed smooth ring R. Particle B hangs in equilibrium vertically below the ring. Particle A moves in horizontal circles with speed v. Particles A and B are at the same horizontal level. The angle between AR and BR is θ (see diagram).

(a) Show that $\cos\theta = \frac{1}{3}$. [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(b) Find an expression for v in terms of a and g. [4]

.....

.....

.....

.....

.....

.....

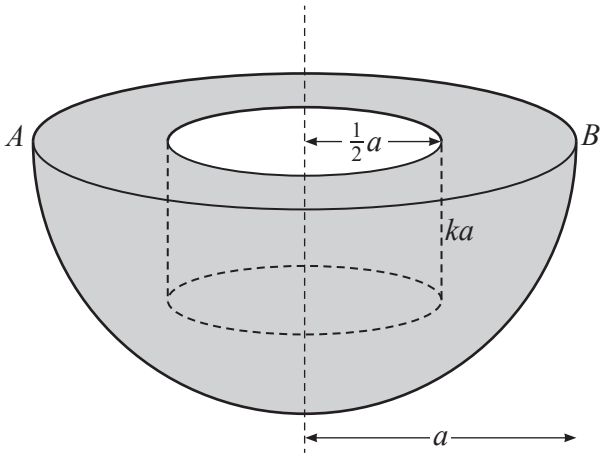
.....

.....

.....

.....

4



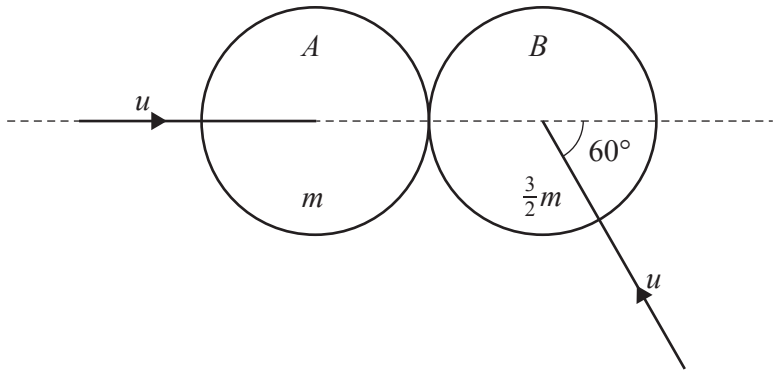
An object is formed by removing a solid cylinder, of height ka and radius $\frac{1}{2}a$, from a uniform solid hemisphere of radius a. The axes of symmetry of the hemisphere and the cylinder coincide and one circular face of the cylinder coincides with the plane face of the hemisphere. AB is a diameter of the circular face of the hemisphere (see diagram).

(a) Show that the distance of the centre of mass of the object from AB is $\dfrac{3a(2-k^2)}{2(8-3k)}$. [4]

When the object is freely suspended from the point A, the line AB makes an angle θ with the downward vertical, where $\tan\theta = \frac{7}{18}$.

(b) Find the possible values of k. [3]

5



Two uniform smooth spheres A and B of equal radii have masses m and $\frac{3}{2}m$ respectively. The two spheres are each moving with speed u on a horizontal surface when they collide. Immediately before the collision A's direction of motion is along the line of centres, and B's direction of motion makes an angle of 60° with the line of centres (see diagram). The coefficient of restitution between the spheres is $\frac{2}{3}$.

(a) Find the angle through which the direction of motion of B is deflected by the collision. [6]

.....

.....

.....

.....

.....

.....

.....

.....

.....

(b) Find the loss in the total kinetic energy of the system as a result of the collision. [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

6 A particle P of mass 2 kg moves along a horizontal straight line. The point O is a fixed point on this line. At time t s the velocity of P is $v\,\text{m}\,\text{s}^{-1}$ and the displacement of P from O is x m.

A force of magnitude $\left(8x - \frac{128}{x^3}\right)$ N acts on P in the direction OP. When $t = 0$, $x = 8$ and $v = -15$.

(a) Show that $v = -\frac{2}{x}(x^2 - 4)$. [5]

(b) Find an expression for x in terms of t. [4]

7 One end of a light inextensible string of length a is attached to a fixed point O. The other end of the string is attached to a particle P of mass m. The particle P is held vertically below O with the string taut and then projected horizontally. When the string makes an angle of 60° with the upward vertical, P becomes detached from the string. In its subsequent motion, P passes through the point A which is a distance a vertically above O.

(a) The speed of P when it becomes detached from the string is V. Use the equation of the trajectory of a projectile to find V in terms of a and g. [4]

(b) Find, in terms of m and g, the tension in the string immediately after P is initially projected horizontally. [4]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

BLANK PAGE

BLANK PAGE

Cambridge Assessment International Education

Cambridge International AS & A Level

FURTHER MATHEMATICS **9231/32**

Paper 3 Further Mechanics **October/November 2021**

MARK SCHEME

Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2021 series for most Cambridge IGCSE™, Cambridge International A and AS Level components and some Cambridge O Level components.

[Turn over

PUBLISHED

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:
Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:
Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:
Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:
Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

PUBLISHED

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1(a)	Velocity: $\rightarrow u\cos\alpha$	**B1**	
	$\uparrow u\sin\alpha - gT$	**B1**	Allow 10 for g. Must be T.
		2	
1(b)	$\dfrac{u\cos\alpha}{u\sin\alpha - gT} = -\dfrac{\sin\alpha}{\cos\alpha}$ oe	**M1 FT**	Allow missing minus sign on RHS for M1. FT from **(a)**.
	$T = \dfrac{u}{g\sin\alpha}$	**A1**	
		2	
1(c)	$\sin\alpha < 1$ giving $T > \dfrac{u}{g}$	**B1**	AG
		1	

Question	Answer	Marks	Guidance
2	At the collision of P and Q: $(m+km)v=kmu$	**M1**	Momentum conserved, allow missing k on RHS.
	So $v=\dfrac{k\sqrt{4ga}}{(1+k)}$	**A1**	
	EPE = $\dfrac{1}{2}\times\dfrac{5mg}{a}\times\left(\dfrac{a}{5}\right)^2 \quad \left(=\dfrac{mga}{10}\right)$	**B1**	
	Loss in KE = Gain in EPE: $\dfrac{1}{2}m(k+1)v^2=\dfrac{1}{2}\times\dfrac{5mg}{a}\times\left(\dfrac{a}{5}\right)^2$	**M1**	Energy equation, LHS correct, EPE dimensionally correct.
	Substitute for v and rearrange to form quadratic equation in k $20k^2=1+k$	**M1**	
	$k=\dfrac{1}{4}$	**A1**	
		6	

Question	Answer	Marks	Guidance
3(a)	$T = 3mg$ and $T\cos\theta = mg$	**M1**	Must see both of these separately.
	Combining, $\cos\theta = \frac{1}{3}$	**A1**	At least one step of working, AG.
		2	
3(b)	$(\cos\theta = \frac{a-x}{x}$, where $x = AR)$ $AR = \frac{3}{4}a$ or $BR = \frac{1}{4}a$ or radius = $\frac{a}{\sqrt{2}}$	**B1**	$\left(\sin\theta = \frac{\sqrt{8}}{3}\right)$
	$T\sin\theta = \frac{mv^2}{r}$	**M1**	
	Combining to find an equation in v^2, a and g only.	**DM1**	
	$v^2 = 2ga$, $v = \sqrt{2ga}$	**A1**	
		4	

Question	Answer	Marks	Guidance
4(a)	<table><tr><th></th><th>Volume</th><th>Centre of mass from AB</th></tr><tr><td>Hemisphere</td><td>$\frac{2}{3}\pi a^3$</td><td>$\frac{3}{8}a$</td></tr><tr><td>Cylinder</td><td>$\pi ka(\frac{a}{2})^2$</td><td>$\frac{ka}{2}$</td></tr><tr><td>Remainder</td><td>$\frac{2}{3}\pi a^3 - \pi ka\left(\frac{a}{2}\right)^2$</td><td>$\bar{x}$</td></tr></table>	**M1**	Attempt at moments, 3 terms.
	Taking moments about AB: $\left(\frac{2}{3}\pi a^3 - \pi ka\left(\frac{a}{2}\right)^2\right) \times \bar{x} = \left(\frac{2}{3}\pi a^3 \times \frac{3}{8}a\right) - \left(\pi ka\left(\frac{a}{2}\right)^2 \times \frac{ka}{2}\right)$	**A1** **A1**	Any 2 terms correct. All correct.
	$\bar{x} = \frac{3a\left(2-k^2\right)}{2(8-3k)}$	**A1**	Shown convincingly, AG.
		4	
4(b)	$\tan\theta = \frac{\bar{x}}{a}$ $\frac{3\left(2-k^2\right)}{2(8-3k)} = \frac{7}{18}$	**B1**	
	$27k^2 - 21k + 2 = 0$	**M1**	Rearrange to form quadratic.
	$k = \frac{2}{3}$ and $k = \frac{1}{9}$	**A1**	Both answers correct.
		3	

Question	Answer	Marks	Guidance
5(a)	Let speeds of A and B along line of centres after collision be v_1 and v_2 $mv_1 + \frac{3}{2}mv_2 = -\frac{3}{2}mu\cos 60° + mu \quad \left(= \frac{u}{4}\right)$	**M1**	Momentum with masses correct.
	$v_2 - v_1 = -\frac{2}{3}(-u\cos 60° - u) \quad (= u)$	**M1**	Restitution, with consistent signs on LHS.
	$\left(v_1 = -\frac{1}{2}u\right) \qquad v_2 = \frac{1}{2}u$	**A1**	
	Perpendicular to line of centres, speed of B is $u\sin 60° = \frac{\sqrt{3}}{2}u$	**B1**	
	Direction of B is now 60° above line of centres.	**M1**	
	Angle of deflection is 60°.	**A1 FT**	FT (120° – *their* direction of B angle)
		6	
5(b)	KE before = $\frac{1}{2}mu^2 + \frac{1}{2}.\frac{3m}{2}u^2 = \frac{5}{4}mu^2$	**B1**	
	KE after = $\frac{1}{2}m\left(\frac{u}{2}\right)^2 + \frac{1}{2}.\frac{3m}{2}\left(\left(\frac{u}{2}\right)^2 + \left(\frac{\sqrt{3}u}{2}\right)^2\right) \quad \left(= \frac{7}{8}mu^2\right)$	**B1 FT**	FT only *their* speeds from **(a)**
	Loss in KE = $\frac{3}{8}mu^2$	**B1**	
		3	

Question	Answer	Marks	Guidance
6(a)	$2v\frac{dv}{dx}=8x-\frac{128}{x^3}$	**M1**	Separate variables and integrate, + c not required for M1.
	$v^2=4x^2+64x^{-2}+c$	**A1**	OE.
	$x=8$, $v=-15$ and $c=-32$	**M1**	Use initial condition.
	$v^2=\frac{4}{x^2}\left(x^4-8x^2+16\right)$ or $4x^2-32+\frac{64}{x^2}$	**A1**	Correct expression for v^2, AEF.
	$v^2=\frac{4}{x^2}\left(x^2-4\right)^2$ giving $v=-\frac{2}{x}\left(x^2-4\right)$	**A1**	Convincingly shown, e.g. v is negative initially, AG.
		5	
6(b)	$\frac{1}{2}\ln(x^2-4)=-2t\left(+A\right)$	**M1**	Use $v=\frac{dx}{dt}$ and integrate.
	$t=0$, $x=8$, $A=\frac{1}{2}\ln 60$	**DM1**	Use initial condition.
	$\frac{1}{2}\ln\left(\frac{x^2-4}{60}\right)=-2t$ giving $\frac{x^2-4}{60}=e^{-4t}$	**M1**	Remove log.
	$x=\sqrt{4+60e^{-4t}}$	**A1**	CAO
		4	

Question	Answer	Marks	Guidance
7(a)	Coordinates of A: $x = a\sin 60, \quad y = a - a\cos 60$	**B1**	
	$\frac{a}{2} = \frac{a\sqrt{3}}{2}\sqrt{3} - \frac{g\frac{\left(a\sqrt{3}\right)^2}{2^2}}{2V^2.\frac{1}{4}}$	**M1**	Substitute *their* (x, y) into correct trajectory equation.
	Rearrange to find V^2.	**M1**	
	$V^2 = \frac{3}{2}ag, \quad V = \sqrt{\frac{3}{2}ag}$	**A1**	
		4	
7(b)	$\frac{1}{2}mu^2 - \frac{1}{2}mV^2 = mga\left(1+\cos 60\right)$	**M1**	Energy equation.
	$u^2 = \frac{9}{2}ag$	**A1**	u is the speed at P.
	$T - mg = \frac{m}{a}u^2$	**M1**	N2L
	$T = \frac{11}{2}mg$	**A1**	
		4	

Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME	
CENTRE NUMBER	
CANDIDATE NUMBER	

FURTHER MATHEMATICS **9231/32**

Paper 3 Further Mechanics **May/June 2021**

1 hour 30 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.
- Where a numerical value for the acceleration due to gravity (g) is needed, use $10\,\text{m}\,\text{s}^{-2}$.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Any blank pages are indicated.

[Turn over

BLANK PAGE

1 A particle P of mass 1 kg is moving along a straight line against a resistive force of magnitude $\frac{10\sqrt{v}}{(t+1)^2}$ N, where $v\,\text{m}\,\text{s}^{-1}$ is the speed of P at time t s. When $t = 0$, $v = 25$.

Find an expression for v in terms of t. [5]

2 A hollow hemispherical bowl of radius a has a smooth inner surface and is fixed with its axis vertical. A particle P of mass m moves in horizontal circles on the inner surface of the bowl, at a height x above the lowest point of the bowl. The speed of P is $\sqrt{\frac{8}{3}ga}$.

Find x in terms of a. [6]

3 One end of a light elastic string, of natural length a and modulus of elasticity kmg, is attached to a fixed point A. The other end of the string is attached to a particle P of mass $4m$. The particle P hangs in equilibrium a distance x vertically below A.

(a) Show that $k = \dfrac{4a}{x-a}$. [1]

.....

.....

.....

.....

An additional particle, of mass $2m$, is now attached to P and the combined particle is released from rest at the original equilibrium position of P. When the combined particle has descended a distance $\frac{1}{3}a$, its speed is $\frac{1}{3}\sqrt{ga}$.

(b) Find x in terms of a. [6]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

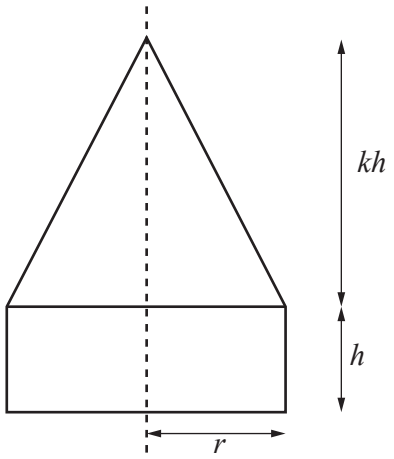
.....

.....

.....

.....

4



A uniform solid circular cone has vertical height kh and radius r. A uniform solid cylinder has height h and radius r. The base of the cone is joined to one of the circular faces of the cylinder so that the axes of symmetry of the two solids coincide (see diagram, which shows a cross-section). The cone and the cylinder are made of the same material.

(a) Show that the distance of the centre of mass of the combined solid from the base of the cylinder is $\frac{h(k^2+4k+6)}{4(3+k)}$. [4]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

The solid is placed on a plane that is inclined to the horizontal at an angle θ. The base of the cylinder is in contact with the plane. The plane is sufficiently rough to prevent sliding. It is given that $3h = 2r$ and that the solid is on the point of toppling when $\tan\theta = \frac{4}{3}$.

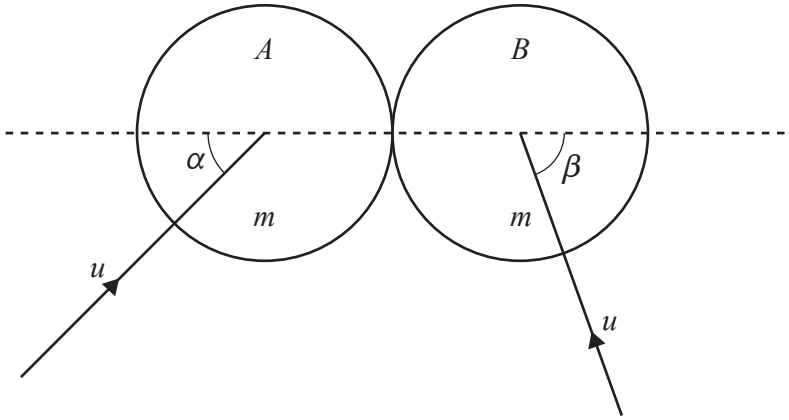
(b) Find the value of k. [3]

5 A particle P of mass m is attached to one end of a light inextensible string of length a. The other end of the string is attached to a fixed point O. The particle completes vertical circles with centre O. The points A and B are on the path of P, both on the same side of the vertical through O. OA makes an angle θ with the downward vertical through O and OB makes an angle θ with the upward vertical through O.

The speed of P when it is at A is u and the speed of P when it is at B is $\sqrt{ag}$. The tensions in the string at A and B are T_A and T_B respectively. It is given that $T_A = 7T_B$.

Find the value of θ and find an expression for u in terms of a and g. [8]

6



Two uniform smooth spheres A and B of equal radii each have mass m. The two spheres are each moving with speed u on a horizontal surface when they collide. Immediately before the collision, A's direction of motion makes an angle α with the line of centres, and B's direction of motion makes an angle β with the line of centres (see diagram). The coefficient of restitution between the spheres is $\frac{1}{3}$ and $2\cos\beta = \cos\alpha$.

(a) Show that the direction of motion of A after the collision is perpendicular to the line of centres. [4]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

The total kinetic energy of the spheres after the collision is $\frac{3}{4}mu^2$.

(b) Find the value of α. [4]

7 A particle P is projected from a point O on a horizontal plane and moves freely under gravity. The initial velocity of P is $100\,\text{m}\,\text{s}^{-1}$ at an angle θ above the horizontal, where $\tan\theta = \frac{4}{3}$. The two times at which P's height above the plane is H m differ by 10 s.

(a) Find the value of H. [5]

(b) Find the magnitude and direction of the velocity of P one second before it strikes the plane. [4]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

BLANK PAGE

BLANK PAGE

Cambridge Assessment International Education

Cambridge International AS & A Level

FURTHER MATHEMATICS **9231/32**

Paper 3 Further Mechanics **May/June 2021**

MARK SCHEME

Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2021 series for most Cambridge IGCSE™, Cambridge International A and AS Level components and some Cambridge O Level components.

This document consists of **13** printed pages.

PUBLISHED

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:
Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:
Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:
Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:
Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

PUBLISHED

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	$\frac{dv}{\sqrt{v}} = -\frac{10dt}{(t+1)^2}$	**M1**	Separate variables.
	$2\sqrt{v} = \frac{10}{t+1} + A$	**M1 A1**	Attempt to integrate.
	$t = 0, \quad v = 25, \quad A = 0$	**M1**	Use correct initial condition.
	$v = \frac{25}{(t+1)^2}$	**A1**	CAO
		5	

Question	Answer	Marks	Guidance
2	$\uparrow R\cos\theta = mg$	**B1**	
	$\rightarrow R\sin\theta = \frac{mv^2}{r}$	**B1**	
	$r = a\sin\theta$	**B1**	
	$8\cos\theta = 3\left(1-\left(\cos\theta\right)^2\right)$	**M1**	Quadratic equation in $\cos\theta$.
	$\cos\theta = \frac{1}{3}$	**A1**	
	$x = \frac{2}{3}a$	**A1**	
		6	

Question	Answer	Marks	Guidance
3(a)	Use Hooke's Law: $4mg = \frac{kmg(x-a)}{a}$ leading to $k = \frac{4a}{x-a}$	**B1**	AG. Shown convincingly.
		1	
3(b)	Gain in KE + gain in EPE = loss in GPE	**B1**	One correct EPE term seen.
	$\frac{1}{2}\times 6m\times\frac{ga}{9}+\frac{1}{2}\frac{kmg}{a}\left(\left(x+\frac{a}{3}-a\right)^2-(x-a)^2\right)=6mg\times\frac{a}{3}$	**M1** **A1**	All 3 types of energy included in energy equation. All terms correct.
	Simplify and substitute for k from part **(a)**	**M1**	
	Obtain linear equation in x and a	**M1**	
	$x=\frac{5}{3}a$	**A1**	($k = 6$)
		6	

Question	Answer	Marks	Guidance
4(a)	<table><tr><th></th><th>Volume</th><th>Centre of mass from base of cylinder</th></tr><tr><td>Cone</td><td>$\frac{1}{3}\pi r^2 kh$</td><td>$h+\frac{kh}{4}$</td></tr><tr><td>Cylinder</td><td>$\pi r^2 h$</td><td>$\frac{h}{2}$</td></tr><tr><td>Combined</td><td>$\pi r^2 h\left(\frac{1}{3}k+1\right)$</td><td>$\bar{x}$</td></tr></table>	**M1**	
	Take moments about base: $\pi r^2 h\left(\frac{1}{3}k+1\right)\bar{x}=\frac{1}{3}\pi r^2 kh\left(h+\frac{kh}{4}\right)+\pi r^2 h\ \frac{h}{2}$	**A1** **A1**	2 terms correct. All terms correct.
	$\bar{x}=\frac{h\left(k^2+4k+6\right)}{4\left(3+k\right)}$	**A1**	AG. Shown convincingly.
		4	
4(b)	$\tan\theta=\frac{r}{\bar{x}}$	**M1**	
	$\frac{4}{3}=\frac{6h\left(k+3\right)}{h\left(k^2+4k+6\right)}$ $2k^2-k-15=0$	**M1**	Equate to $\frac{4}{3}$ and simplify to quadratic.
	$k=3$	**A1**	CAO. No other solutions.
		3	

Question	Answer	Marks	Guidance
5	$T_A - mg\cos\theta = \frac{mu^2}{a}$	**B1**	
	$T_B + mg\cos\theta = \frac{mag}{a}$	**B1**	
	$T_A = 7T_B$ so $mg\cos\theta + \frac{mu^2}{a} = 7\left(-mg\cos\theta + \frac{mag}{a}\right)$ $u^2 = ag(7 - 8\cos\theta)$	**M1**	Use given relationship and combine.
	Energy: $\frac{1}{2}mu^2 - \frac{1}{2}mag = mg(a\cos\theta + a\cos\theta)$ So $u^2 = ag(4\cos\theta + 1)$	**M1 A1**	Energy equation.
	Equate expressions for u^2	**M1**	
	$\cos\theta = \frac{1}{2}$, $\theta = 60°$	**A1**	CAO
	$u = \sqrt{3ga}$	**A1**	CAO
		8	

Question	Answer	Marks	Guidance
6(a)	Along line of centres, speeds v_1 and v_2 $mv_1 + mv_2 = mu\cos\alpha - mu\cos\beta$	**M1**	Momentum (condone missing masses).
	$v_2 - v_1 = eu(\cos\beta + \cos\alpha)$	**M1**	Restitution.
	Both correct, masses seen.	**A1**	
	$v_1 = 0$ so A has no speed along line of centres: moves perpendicular to line of centres	**A1**	AG.
		4	
6(b)	$(v_2 = \frac{1}{2}u\ \cos\alpha = u\cos\beta\ )$ KE of B after collision is $\frac{1}{2}m\left(v_2{}^2 + (u\sin\beta)^2\right)$ KE of A after collision = $\frac{1}{2}m(u\sin\alpha)^2$	**M1**	Both components.
	Add both KEs and equate to $\frac{3}{4}mu^2$	**M1**	
	Simplify to equation in $\sin\alpha$	**M1**	
	$\sin\alpha = \frac{1}{\sqrt{2}}$, $\alpha = 45°$	**A1**	
		4	

Question	Answer	Marks	Guidance
7(a)	At greatest height $0 = 100\sin\theta - gt$	**M1**	
	$t = 8$	**A1**	
	Therefore times at height H are $t = 3$ (and $t = 13$)	**B1**	
	Substitute into $H = 100\sin\theta t - \frac{1}{2}gt^2$	**M1**	
	$H = 195$	**A1**	
	Alternative method to question **7(a)**		
	$\uparrow H = 100\sin\theta t - \frac{1}{2}gt^2$	**M1**	
	And $H = 100\sin\theta(t+10) - \frac{1}{2}g(t+10)^2$	**A1**	
	Subtract: $1000\sin\theta = \frac{1}{2}g(20t+100)$	**M1**	
	$t = 3$	**B1**	
	$H = 195$	**A1**	

Question	Answer	Marks	Guidance
7(a)	Alternative method to question **7(a)**		
	$\uparrow H = 100\sin\theta t - \frac{1}{2}gt^2$	**B1**	
	Difference between roots = $\frac{\sqrt{(100\sin\theta)^2 - 2gH}}{\frac{1}{2}g}$	**M1 A1**	
	Equate to 10 and rearrange to find H	**M1**	
	$H = 195$	**A1**	
		5	
7(b)	Time to required point = 15 s	**B1**	
	$\uparrow v = 100\sin\theta - 10\times 15 (= -70)$ $\rightarrow v = 100\cos\theta = 60$	**B1**	Both components.
	Magnitude = 92.2	**B1**	
	Angle below horizontal = $\tan^{-1}(70/60) = 49.4°$	**B1**	
		4	

Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER | | | | | |

CANDIDATE NUMBER | | | | |

6007163164

FURTHER MATHEMATICS **9231/33**

Paper 3 Further Mechanics **May/June 2021**

1 hour 30 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

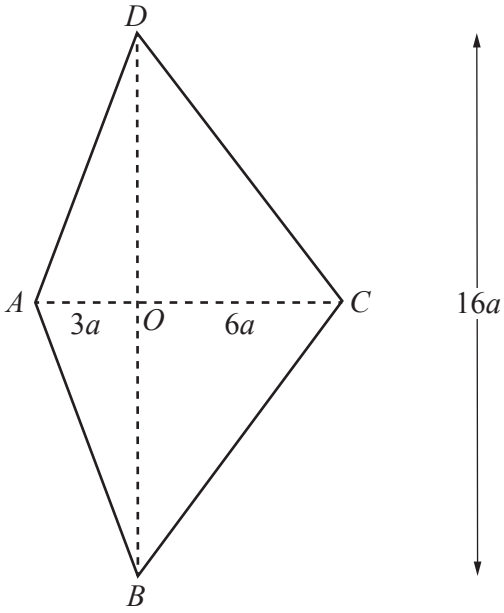
- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.
- Where a numerical value for the acceleration due to gravity (g) is needed, use $10\,\text{m}\,\text{s}^{-2}$.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Any blank pages are indicated.

1



A uniform lamina $ABCD$ consists of two isosceles triangles ABD and BCD. The diagonals of $ABCD$ meet at the point O. The length of AO is $3a$, the length of OC is $6a$ and the length of BD is $16a$ (see diagram).

Find the distance of the centre of mass of the lamina from DB. [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

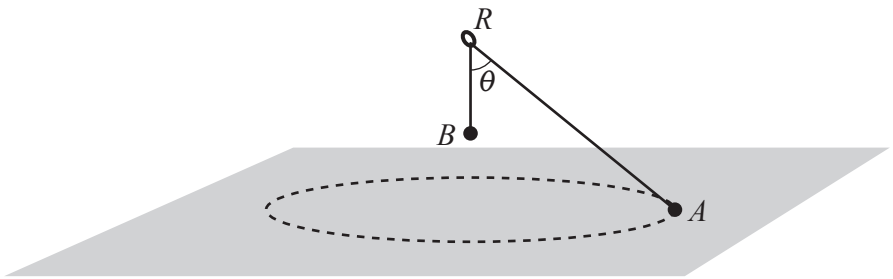
.....

.....

2 One end of a light elastic string of natural length 0.8 m and modulus of elasticity 36 N is attached to a fixed point O on a smooth plane. The plane is inclined at an angle α to the horizontal, where $\sin\alpha = \frac{3}{5}$. A particle P of mass 2 kg is attached to the other end of the string. The string lies along a line of greatest slope of the plane with the particle below the level of O. The particle is projected with speed $\sqrt{2}\,\text{m s}^{-1}$ directly down the plane from the position where OP is equal to the natural length of the string.

Find the maximum extension of the string during the subsequent motion. [5]

3

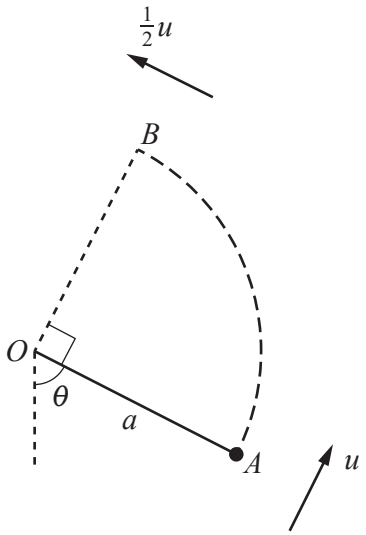


Particles A and B, of masses $3m$ and m respectively, are connected by a light inextensible string of length a that passes through a fixed smooth ring R. Particle B hangs in equilibrium vertically below the ring. Particle A moves in horizontal circles on a smooth horizontal surface with speed $\frac{2}{5}\sqrt{ga}$. The angle between AR and BR is θ (see diagram). The normal reaction between A and the surface is $\frac{12}{5}mg$.

(a) Find $\cos\theta$. [3]

(b) Find, in terms of a, the distance of B below the ring. [3]

4



A particle of mass m is attached to one end of a light inextensible string of length a. The other end of the string is attached to a fixed point O. The particle is initially held with the string taut at the point A, where OA makes an angle θ with the downward vertical through O. The particle is then projected with speed u perpendicular to OA and begins to move upwards in part of a vertical circle. The string goes slack when the particle is at the point B where angle AOB is a right angle. The speed of the particle when it is at B is $\frac{1}{2}u$ (see diagram).

Find the tension in the string at A, giving your answer in terms of m and g. [8]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

5 A particle P of mass m kg is projected vertically upwards from a point O, with speed $20\,\text{m s}^{-1}$, and moves under gravity. There is a resistive force of magnitude $2mv$ N, where $v\,\text{m s}^{-1}$ is the speed of P at time t s after projection.

(a) Find an expression for v in terms of t, while P is moving upwards. [6]

The displacement of P from O is x m at time t s.

(b) Find an expression for x in terms of t, while P is moving upwards. [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(c) Find, correct to 3 significant figures, the greatest height above O reached by P. [2]

.....

.....

.....

.....

.....

.....

.....

.....

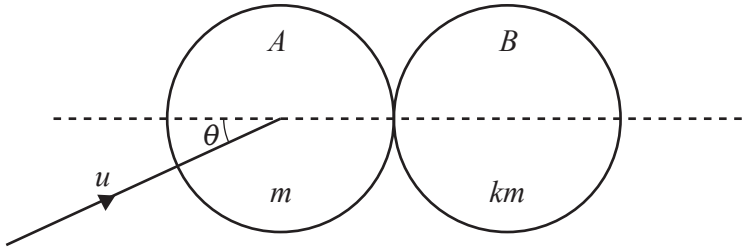
.....

.....

.....

.....

6



Two uniform smooth spheres A and B of equal radii have masses m and km respectively. Sphere A is moving with speed u on a smooth horizontal surface when it collides with sphere B which is at rest. Immediately before the collision, A's direction of motion makes an angle θ with the line of centres (see diagram). The coefficient of restitution between the spheres is $\frac{1}{3}$.

(a) Show that the speed of B after the collision is $\dfrac{4u\cos\theta}{3(1+k)}$. [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

70% of the total kinetic energy of the spheres is lost as a result of the collision.

(b) Given that $\tan\theta = \frac{1}{3}$, find the value of k. [6]

7 A particle P is projected with speed u at an angle θ above the horizontal from a point O on a horizontal plane and moves freely under gravity. The horizontal and vertical displacements of P from O at a subsequent time t are denoted by x and y respectively.

(a) Use the equation of the trajectory given in the List of formulae (MF19), together with the condition $y = 0$, to establish an expression for the range R in terms of u, θ and g. [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(b) Deduce an expression for the maximum height H, in terms of u, θ and g. [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

It is given that $R = \frac{4H}{\sqrt{3}}$.

(c) Show that $\theta = 60°$. [1]

.....

.....

.....

.....

.....

.....

It is given also that $u = \sqrt{40}\,\text{m s}^{-1}$.

(d) Find, by differentiating the equation of the trajectory or otherwise, the set of values of x for which the direction of motion makes an angle of less than 45° with the horizontal. [4]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

BLANK PAGE

BLANK PAGE

Cambridge Assessment International Education

Cambridge International AS & A Level

FURTHER MATHEMATICS **9231/33**

Paper 3 Further Mechanics **May/June 2021**

MARK SCHEME

Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2021 series for most Cambridge IGCSE™, Cambridge International A and AS Level components and some Cambridge O Level components.

[Turn over

PUBLISHED

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:
Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:
Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:
Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:
Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

PUBLISHED

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	<table><tr><th></th><th>Area</th><th>Centre of mass from DB</th></tr><tr><td>ABD</td><td>$24a^2$</td><td>$-a$</td></tr><tr><td>BCD</td><td>$48a^2$</td><td>$2a$</td></tr><tr><td>Combined</td><td>$72a^2$</td><td>$\bar{x}$</td></tr></table>	**B1**	All distances correct. $ABCD$ can be split in other ways, for example ADC and ABC.
	Taking moments about DB: $72a^2\bar{x} = 24a^2 \times -a + 48a^2 \times 2a$ *OR* Taking moments about A: $72a^2\bar{x} = 24a^2 \times 2a + 48a^2 \times 5a$ *OR* Taking moments about G: $24a^2(\bar{x}+a) = 48a^2 \times (2a-\bar{x})$	**M1**	Moments equation with masses in correct ratio.
	$\bar{x} = a$	**A1**	CWO
	Alternative method for question 1		
	ADC: distance of centre of mass from BD = $\frac{6a-3a}{3} = a$ ABC: distance of centre of mass from BD = $\frac{6a-3a}{3} = a$	**B1**	One calculation.
	Second calculation or statement about symmetry	**M1**	
	$\bar{x} = a$	**A1**	
		3	

Question	Answer	Marks	Guidance
2	$\frac{1}{2}\times\frac{36x^2}{0.8}$ $(=22.5x^2)$	**B1**	EPE correct.
	Loss in GPE + loss in KE = gain in EPE $x\,mg\sin\alpha+\frac{1}{2}m\times 2=\frac{1}{2}\times\frac{36x^2}{0.8}$	***M1**	Energy equation with only GPE, EPE and KE terms, allow sign errors, allow missing g for M1 only, weight must be resolved (allow sin or cos).
	All terms correct	**A1**	
	$\left(\frac{6}{5}x+\frac{1}{5}=\frac{9}{4}x^2\right)$ [leading to $45x^2-24x-4=0$]	**DM1**	Simplify to 3-term quadratic and attempt to solve.
	$(3x-2)(15x+2)=0$ $x=\frac{2}{3}$ only	**A1**	
		5	

Question	Answer	Marks	Guidance
3(a)	For B: $T = mg$	**B1**	May be embedded.
	For A: $R + T\cos\theta = 3mg$	**M1**	All 3 terms, allow sign errors, allow sin/cos mix.
	Use given R to obtain $\cos\theta = \frac{3}{5}$	**A1**	
		3	
3(b)	$T\sin\theta = \frac{3mv^2}{r}$	**M1**	May be seen in part **(a)**, allow sin/cos mix.
	$r = AR\sin\theta$	**B1**	Or equivalent.
	[Combine to give $AR = \frac{3a}{4}$, so] $BR = \frac{1}{4}a$	**A1**	
		3	

Question	Answer	Marks	Guidance
4	$\frac{1}{2}mu^2 - \frac{1}{2}m\left(\frac{u}{2}\right)^2 = mg\left(a\cos\theta + a\sin\theta\right)$	*M1	Energy equation, with 2 KE terms and a two-part GPE term, allow cos/sin mix.
	$\frac{3}{4}u^2 = 2ag\left(\cos\theta + \sin\theta\right)$	A1	
	At B, tension in string is zero, so $mg\sin\theta = \frac{m\left(\frac{u}{2}\right)^2}{a}$ ($u^2 = 4ag\sin\theta$)	B1	N2L
	Eliminate u^2	DM1	
	$\tan\theta = 2$ OE	A1	
	At A, $T - mg\cos\theta = \frac{mu^2}{a}$	B1	N2L
	$T = \frac{9\sqrt{5}}{5}mg$ $(= 4.02mg)$	M1 A1	Substitute to find T.
	Alternative method for question 4		
	$\frac{1}{2}mu^2 - \frac{1}{2}m\left(\frac{u}{2}\right)^2 = mg\left(a\cos\theta + a\sin\theta\right)$	*M1	Energy equation, with 2 KE terms and a two-part GPE term, allow cos/sin mix.
	$\frac{3}{4}u^2 = 2ag\left(\cos\theta + \sin\theta\right)$	A1	

Question	Answer	Marks	Guidance
4	At B, tension in string is zero, so $mg\sin\theta = \dfrac{m\left(\frac{u}{2}\right)^2}{a}$ $(u^2 = 4ag\sin\theta)$	**B1**	N2L
	Eliminate θ: $u^2 = \dfrac{8ag\sqrt{5}}{5}$	**DM1 A1**	
	At A, $T - mg\cos\theta = \dfrac{mu^2}{a}$	**B1**	N2L
	$T = \dfrac{9\sqrt{5}}{5}mg \quad (= 4.02mg)$	**M1 A1**	Substitute to find T.
		8	

Question	Answer	Marks	Guidance
5(a)	$m\frac{dv}{dt}=-mg-2mv$	**B1**	Use of SUVAT implies 0 marks. N2L, must include m .
	$\ln(5+v)=-2t(+A)$	**M1**	Separate variables and integrate 3-term N2L, condone omission of constant.
	$\ln(5+v)=-2t+A$	**A1 FT**	FT only sign error in N2L.
	$t=0, v=20,\ \ A=\ln 25$	**M1**	Use correct initial condition.
	$2t=\ln\left(\frac{25}{5+v}\right)$, $e^{2t}=\frac{25}{5+v}$	**M1**	Remove all logs.
	$v=25e^{-2t}-5$	**A1**	
	Alternative method for question 5(a)		
	$m\frac{dv}{dt}=-mg-2mv$	**B1**	N2L, must include m .
	$\frac{dv}{dt}+2v=-g$: Integrating factor = e^{2t}	**M1**	
	$\frac{d\left(ve^{2t}\right)}{dt}=-ge^{2t}$, $ve^{2t}=-\frac{g}{2}e^{2t}(+A)$	**M1**	Integrate both sides, condone omission of constant.
	$ve^{2t}=-\frac{g}{2}e^{2t}+\mathrm{A}$	**A1 FT**	FT only sign error in N2L.

Question	Answer	Marks	Guidance
5(a)	$t=0, v=20, \quad A=25$	**M1**	Use correct initial condition.
	$ve^{2t} = -\frac{g}{2}e^{2t} + 25, \quad v = 25e^{-2t} - 5$	**A1**	
		6	
5(b)	$x = -\frac{25}{2}e^{-2t} - 5t(+B)$	**M1**	Use of SUVAT implies 0 marks. Integrate their expression from part **(a)**.
	$t=0, x=0, \quad B=\frac{25}{2}$ $x = \frac{25}{2}\left(1-e^{-2t}\right) - 5t$	**A1 FT**	FT only expressions of the form $v = Pe^{kt} + Q$ for P, Q non-zero.
		2	
5(c)	Greatest height when $v = 0$, so $t = 0.8047\ldots$ or $\frac{1}{2}\ln 5$	**M1**	Use of SUVAT in part **(a)** or part **(b)** implies 0 marks. Find value of t, may be embedded.
	$x = 5.98$ m	**A1**	CWO
		2	

Question	Answer	Marks	Guidance
6(a)	Let velocities of A and B along line of centres after collision be v_1 and v_2. $mv_1 + kmv_2 = mu\cos\theta$.	**M1**	Momentum, must include m, allow cos/sin mix.
	$v_2 - v_1 = \frac{1}{3}u\cos\theta$	**M1**	Restitution, consistent signs, correct way up.
	Solve: $v_2 = \frac{4u\cos\theta}{3(1+k)}$	**A1**	AG shown convincingly.
		3	
6(b)	$v_1 = \frac{(3-k)u\cos\theta}{3(1+k)}$	**B1**	Or equivalent, may be unsimplified.
	Use velocity of A with both components.	**B1**	$v_1^2 + (u\sin\theta)^2$ seen.
	$\frac{1}{2}kmv_2^2 + \frac{1}{2}m\left(v_1^2 + (u\sin\theta)^2\right) = \frac{3}{10}\times\frac{1}{2}mu^2$	**M1**	KE after = 30% KE before (all terms present). M0 if incorrect masses.
	Substitute from part **(a)** and for θ.	**M1**	Eliminate trigonometric terms, must be KE equation, in terms of k only.
	$(3-k)^2 + 16k = 2(1+k)^2$, $k^2 - 6k - 7 = 0$	**M1**	Obtain simplified quadratic equation in k.
	$k = 7$	**A1**	
		6	

Question	Answer	Marks	Guidance
7(a)	$y = 0$ in trajectory equation: $R\tan\theta - g\dfrac{R^2}{2u^2(\cos\theta)^2} = 0$	**M1**	
	$(R =)\dfrac{2u^2\sin\theta\cos\theta}{g}$ only	**A1**	Any equivalent single term expression, for example: $\dfrac{u^2\sin 2\theta}{g}$, $\dfrac{2u^2\tan\theta}{g\sec^2\theta}$, at least one intermediate line of working, not just quoting a result. SC B1 using SUVAT.
		2	
7(b)	$x = their\dfrac{u^2\sin\theta\cos\theta}{g}$ and substitute in trajectory equation.	**M1**	Or use SUVAT.
	$H = \dfrac{u^2(\sin\theta)^2}{2g}$	**A1**	Single term.
		2	
7(c)	Use $R = \dfrac{4H}{\sqrt{3}}$ and simplify: $\tan\theta = \sqrt{3}$, $\theta = 60°$	**B1**	AG
		1	

Question	Answer	Marks	Guidance
7(d)	$\frac{dy}{dx} = \tan\theta - \frac{gx}{u^2(\cos\theta)^2}$	**M1**	Differentiate with respect to x.
	$\tan\theta - \frac{x}{4(\cos\theta)^2} = \pm 1$ used	**M1**	Use $\frac{dy}{dx} = \pm 1$ as limiting case.
	$x = \sqrt{3}+1$, $x = \sqrt{3}-1$	**A1**	
	$\sqrt{3}-1 < x < \sqrt{3}+1$	**A1**	Strict inequality, exact values.
	Alternative method for question 7(d)		
	$y = \sqrt{3}x - \frac{1}{2}x^2$, $\frac{dy}{dx} = \sqrt{3} - x$	**M1**	Differentiate with respect to x.
	$\frac{dy}{dx} = \pm 1$ used	**M1**	Use $\frac{dy}{dx} = \pm 1$ as limiting case.
	$x = \sqrt{3}+1$, $x = \sqrt{3}-1$	**A1**	
	$\sqrt{3}-1 < x < \sqrt{3}+1$	**A1**	Strict inequality, exact values.

Question	Answer	Marks	Guidance
7(d)	**Alternative method for question 7(d)**		
	When moving at 45° to horizontal, $v_x = \pm v_y$	**M1**	Used, both cases considered.
	$v_x = \sqrt{40}\cos\theta$, $v_y = \sqrt{40}\sin\theta - 10t$ $t = \frac{1}{10}\left(\sqrt{30} - \sqrt{10}\right)$, $t = \frac{1}{10}\left(\sqrt{30} + \sqrt{10}\right)$	**M1**	
	$x = \sqrt{3} + 1$, $x = \sqrt{3} - 1$	**A1**	
	$\sqrt{3} - 1 < x < \sqrt{3} + 1$	**A1**	Strict inequality, exact values.
		4	