Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME	
CENTRE NUMBER	
CANDIDATE NUMBER	

FURTHER MATHEMATICS **9231/31**

Paper 3 Further Mechanics **October/November 2020**

1 hour 30 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.
- Where a numerical value for the acceleration due to gravity (g) is needed, use $10\,\text{m}\,\text{s}^{-2}$.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Blank pages are indicated.

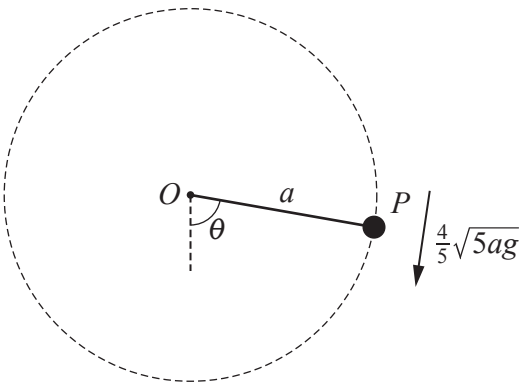
[Turn over

BLANK PAGE

1 A particle P of mass m is placed on a fixed smooth plane which is inclined at an angle θ to the horizontal. A light spring, of natural length a and modulus of elasticity $3mg$, has one end attached to P and the other end attached to a fixed point O at the top of the plane. The spring lies along a line of greatest slope of the plane. The system is released from rest with the spring at its natural length.

Find, in terms of a and θ, an expression for the greatest extension of the spring in the subsequent motion. [3]

2



A particle P is attached to one end of a light inextensible string of length a. The other end of the string is attached to a fixed point O. The particle P is held with the string taut and making an angle θ with the downward vertical. The particle P is then projected with speed $\frac{4}{5}\sqrt{5ag}$ perpendicular to the string and just completes a vertical circle (see diagram).

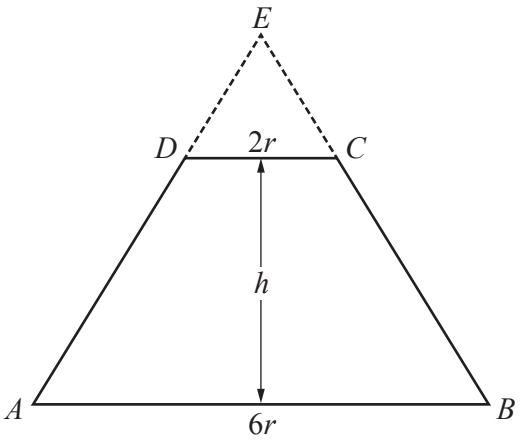
Find the value of $\cos\theta$. [5]

3 One end of a light elastic string, of natural length a and modulus of elasticity $4mg$, is attached to a fixed point O. The other end of the string is attached to a particle of mass m. The particle moves in a horizontal circle with a constant angular speed $\sqrt{\frac{g}{a}}$ with the string inclined at an angle θ to the downward vertical through O. The length of the string during this motion is $(k+1)a$.

(a) Find the value of k. [4]

(b) Find the value of $\cos\theta$. [2]

4



The diagram shows the cross-section $ABCD$ of a uniform solid object which is formed by removing a cone with cross-section DCE from the top of a larger cone with cross-section ABE. The perpendicular distance between AB and DC is h, the diameter AB is $6r$ and the diameter DC is $2r$.

(a) Find an expression, in terms of h, for the distance of the centre of mass of the solid object from AB. [4]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

The object is freely suspended from the point B and hangs in equilibrium. The angle between AB and the downward vertical through B is θ.

(b) Given that $h = \frac{13}{4}r$, find the value of $\tan\theta$. [2]

5 A particle P is projected with speed u at an angle α above the horizontal from a point O on a horizontal plane and moves freely under gravity. The horizontal and vertical displacements of P from O at a subsequent time t are denoted by x and y respectively.

(a) Derive the equation of the trajectory of P in the form

$$y = x\tan\alpha - \frac{gx^2}{2u^2}\sec^2\alpha.$$ [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

The point Q is the highest point on the trajectory of P in the case where $\alpha = 45°$.

(b) Show that the x-coordinate of Q is $\frac{u^2}{2g}$. [3]

.....

.....

.....

.....

.....

.....

.....

(c) Find the other value of α for which P would pass through the point Q. [4]

6 Two smooth spheres A and B have equal radii and masses m and $2m$ respectively. Sphere B is at rest on a smooth horizontal floor. Sphere A is moving on the floor with velocity u and collides directly with B. The coefficient of restitution between the spheres is e.

(a) Find, in terms of u and e, the velocities of A and B after the collision. [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

Subsequently, B collides with a fixed vertical wall which makes an angle θ with the direction of motion of B, where $\tan\theta = \frac{3}{4}$.

The coefficient of restitution between B and the wall is $\frac{2}{3}$. Immediately after B collides with the wall, the kinetic energy of A is $\frac{5}{32}$ of the kinetic energy of B.

(b) Find the possible values of e. [7]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

7 A particle P moving in a straight line has displacement x m from a fixed point O on the line at time t s. The acceleration of P, in m s^{-2}, is given by $\frac{200}{x^2} - \frac{100}{x^3}$ for $x > 0$. When $t = 0$, $x = 1$ and P has velocity $10\,\text{m s}^{-1}$ directed towards O.

(a) Show that the velocity $v\,\text{m s}^{-1}$ of P is given by $v = \frac{10(1-2x)}{x}$. [5]

(b) Show that x and t are related by the equation $e^{-40t} = (2x-1)e^{2x-2}$ and deduce what happens to x as t becomes large. [5]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

BLANK PAGE

BLANK PAGE

Cambridge Assessment International Education

Cambridge International AS & A Level

FURTHER MATHEMATICS **9231/31**

Paper 3 Further Mechanics 31 **October/November 2020**

MARK SCHEME

Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2020 series for most Cambridge IGCSE™, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

This document consists of **14** printed pages.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:
Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:
Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:
Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:
Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	Gain in EPE = $\frac{1}{2}.\frac{3mgx^2}{a}$	**B1**	EPE gain.
	Loss in GPE = $mgx\sin\theta$ Equate	**M1**	Equate energies
	$x=\frac{2}{3}a\sin\theta$	**A1**	Using forces scores B0M0A0
		3	

Question	Answer	Marks	Guidance
2	At top, tension = 0, so $mg=\frac{mv^2}{a}$ $(v^2=ag)$	**B1**	
	$\frac{1}{2}mv^2=\frac{1}{2}mu^2-mga(1+\cos\theta)$	**M1 A1**	Energy equation
	Substitute for u and v: $ag=\frac{16}{25}.5ag-2ag(1+\cos\theta)$	**M1**	Eliminate
	$\cos\theta=\frac{1}{10}$	**A1**	
		5	

Question	Answer	Marks	Guidance
3(a)	$T = 4mg.\frac{ka}{a}$	**B1**	Use Hooke's law
	$T\sin\theta = \left(\frac{mrg}{a}\right) = m(k+1)a\sin\theta.\frac{g}{a}$	**M1**	N2L horizontally. Must see T and k.
	$T = mg(k+1)$	**A1**	
	Equate: $k = \frac{1}{3}$	**A1**	
		4	
3(b)	$\uparrow T\cos\theta = mg$	**M1**	
	$(T = \frac{4}{3}mg)$ $\cos\theta = \frac{mg}{\frac{4}{3}mg} = \frac{3}{4}$	**A1**	
		2	

Question	Answer	Marks	Guidance
4(a)	<table><tr><th></th><th>Volume</th><th>Centre of mass from AB</th></tr><tr><td>Small cone</td><td>$\frac{1}{3}\pi r^2.\frac{h}{2}$</td><td>$h+\frac{1}{4}.\frac{h}{2}\left(=\frac{9h}{8}\right)$</td></tr><tr><td>Large cone</td><td>$\frac{1}{3}\pi(3r)^2.\frac{3h}{2}$</td><td>$\frac{1}{4}.\frac{3h}{2}\left(=\frac{3h}{8}\right)$</td></tr><tr><td>Object</td><td>$\frac{26}{6}\pi(r)^2 h$</td><td>$\bar{x}$</td></tr></table>	**B1**	For 9h/8 or 3h/8 (unsimplified)
	Take moments about AB $\frac{13}{3}\pi r^2 h.\bar{x}=\frac{27}{6}\pi r^2 h.\frac{3h}{8}-\frac{1}{6}\pi r^2 h.\frac{9h}{8}$	**M1 A1**	Moments equation: Allow use of relative masses 1, 26, 27
	$\bar{x}=\frac{9h}{26}$	**A1**	
		4	
4(b)	$\tan\theta=\frac{\bar{x}}{3r}$	**M1**	
	$(=\frac{3h}{26r})$ Use $h=\frac{13}{4}r$ $\tan\theta=\frac{3}{8}$	**A1**	
		2	

Question	Answer	Marks	Guidance
5(a)	$\rightarrow x = u\cos\alpha t$ $\uparrow y = u\sin\alpha t - \frac{1}{2}gt^2$	**B1**	Both
	Eliminate t: $y = u\sin\alpha.\frac{x}{u\cos\alpha} - \frac{1}{2}g\left(\frac{x}{u\cos\alpha}\right)^2$	**M1**	Eliminate
	$y = x\tan\alpha - \frac{gx^2}{2u^2}\sec^2\alpha$	**A1**	AG
		3	
5(b)	Greatest height $= \frac{(u\sin\alpha)^2}{2g} = \frac{u^2}{4g}$	**M1 A1**	Accept alternative methods, for example differentiate expression in **(a)** and equate to 0.
	$t = u\sin 45/g$ so $d = u\cos 45.u\sin 45/g = \frac{u^2}{2g}$	**A1**	AG
		3	

Question	Answer	Marks	Guidance
5(c)	Use greatest height displacements in trajectory equation $\frac{u^2}{4g} = \frac{u^2}{2g}\tan\alpha - \frac{gu^4}{2u^2 4g^2}\sec^2\alpha$	**M1**	Use equation of trajectory (substitute coordinates of Q
	$u^2 = 2u^2\tan\alpha - \frac{u^2}{2}(1+\tan^2\alpha)$	**M1**	Use of $\sec^2\alpha = (1+\tan^2\alpha)$
	$\tan^2\alpha - 4\tan\alpha + 3 = 0$	**M1**	Obtain a three-term quadratic in $\tan\alpha$
	$\tan\alpha = 1,\ 3$ so $\alpha = 71.6°$	**A1**	Both solutions needed
		4	

Question	Answer	Marks	Guidance
6(a)	$mu = mw + 2mv$	**B1**	Momentum equation (with m)
	$v - w = eu$	**B1**	Restitution with consistent signs
	$v = \frac{u}{3}(e+1)$ $w = \frac{u}{3}(1-2e)$	**B1**	Both correct.
		3	

Question	Answer	Marks	Guidance
6(b)	Perpendicular to plane: $y = ev\sin\theta$ Parallel to plane: $x = v\cos\theta$	**B1**	Both
	Speed of $B = \sqrt{x^2 + y^2} = \sqrt{v^2\left(\left(\frac{4}{5}\right)^2 + \left(\frac{2}{3}.\frac{3}{5}\right)^2\right)}$ $\left(= \frac{2}{\sqrt{5}}v\right)$	**M1**	Speed of B
	KE of $B = \frac{1}{2}.2m\frac{4}{5}.\frac{u^2}{9}(e+1)^2$	**M1**	KE of B in terms of u. $\frac{1}{2}$ and $2m$ needed
	KE of $A = \frac{1}{2}.m.\frac{u^2}{9}(1-2e)^2$ So $\frac{1}{2}.m.\frac{u^2}{9}(1-2e)^2 = \frac{5}{32}.\frac{1}{2}.2m\frac{4}{5}.\frac{u^2}{9}(e+1)^2$	**M1 A1**	Relate the two KEs
	$4(1-2e)^2 = (e+1)^2$ or $15e^2 - 18e + 3 = 0$	**M1**	Rearrange and simplify to quadratic
	$1+e = \pm 2(1-2e)$ $e = \frac{1}{5}$, 1	**A1**	Both values
		7	

Question	Answer	Marks	Guidance
7(a)	$v\frac{\mathrm{d}v}{\mathrm{d}x} = -\frac{100}{x^3} + \frac{200}{x^2}$ $\frac{v^2}{2} = \frac{50}{x^2} - \frac{200}{x} + A$	**M1 A1**	Correct equation and attempt to integrate Correct
	$x = 1, v = -10$: $A = 200$	**M1**	Use initial condition
	$v^2 = \frac{100(2x-1)^2}{x^2}$	**M1**	Rearrange to find v^2
	$v = \pm\frac{10(2x-1)}{x}$ and take negative sign to meet initial condition, so $v = \frac{10(1-2x)}{x}$	**A1**	Convincingly shown (no mention of ± scores A0) AG
		5	

Question	Answer	Marks	Guidance
7(b)	$\frac{x\mathrm{d}x}{1-2x}=10\mathrm{d}t$ $\frac{1}{2}\left(\frac{1}{1-2x}-1\right)\mathrm{d}x=10\mathrm{d}t$ $-\frac{1}{4}\ln\lvert 1-2x\rvert-\frac{x}{2}=10t+B$	**M1 A1**	Rearrange and attempt to integrate
	$t=0, x=1:\quad B=-\frac{1}{2}$	**M1**	Use initial condition
	$2x-2=-40t-\ln(\lvert 1-2x\rvert)$ so $e^{-40t}=(2x-1)e^{2x-2}$	**A1**	Convincingly shown, working required AG
	For large values of t, $x\to\frac{1}{2}$	**B1**	CAO
		5	

Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER | | | | | |

CANDIDATE NUMBER | | | | |

FURTHER MATHEMATICS **9231/32**

Paper 3 Further Mechanics **October/November 2020**

1 hour 30 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.
- Where a numerical value for the acceleration due to gravity (g) is needed, use $10\,\text{m}\,\text{s}^{-2}$.

INFORMATION

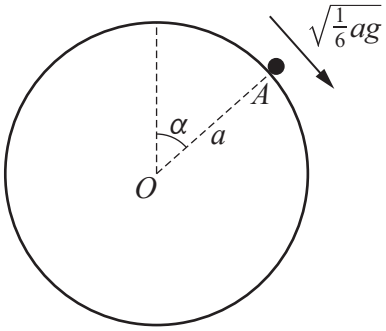
- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Blank pages are indicated.

DC (LK/SG) 187875/2

[Turn over

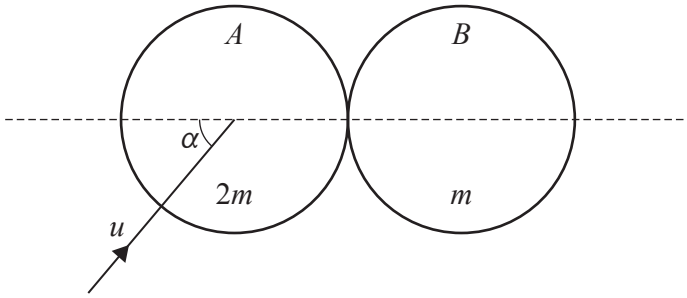
1



A fixed smooth solid sphere has centre O and radius a. A particle of mass m is projected downwards with speed $\sqrt{\frac{1}{6}ag}$ from the point A on the surface of the sphere, where OA makes an angle α with the upward vertical through O (see diagram). The particle moves in part of a vertical circle on the surface of the sphere. It loses contact with the sphere at the point B, where OB makes an angle β with the upward vertical through O.

Given that $\cos\alpha = \frac{2}{3}$, find the value of $\cos\beta$. [5]

2



Two uniform smooth spheres A and B of equal radii have masses $2m$ and m respectively. Sphere B is at rest on a smooth horizontal surface. Sphere A is moving on the surface with speed u and collides with B. Immediately before the collision, the direction of motion of A makes an angle α with the line of centres of the spheres, where $\tan\alpha = \frac{4}{3}$ (see diagram). The coefficient of restitution between the spheres is $\frac{1}{3}$.

Find the speed of A after the collision. [5]

3 An object consists of a uniform solid circular cone, of vertical height $4r$ and radius $3r$, and a uniform solid cylinder, of height $4r$ and radius $3r$. The circular base of the cone and one of the circular faces of the cylinder are joined together so that they coincide. The cone and the cylinder are made of the same material.

(a) Find the distance of the centre of mass of the object from the end of the cylinder that is not attached to the cone. [4]

(b) Show that the object can rest in equilibrium with the curved surface of the cone in contact with a horizontal surface. [3]

4 A particle P of mass m is moving in a horizontal circle with angular speed ω on the smooth inner surface of a hemispherical shell of radius r. The angle between the vertical and the normal reaction of the surface on P is θ.

(a) Show that $\cos\theta = \dfrac{g}{\omega^2 r}$. [3]

The plane of the circular motion is at a height x above the lowest point of the shell. When the angular speed is doubled, the plane of the motion is at a height $4x$ above the lowest point of the shell.

(b) Find x in terms of r. [4]

5 A particle P is projected with speed $u\,\text{m s}^{-1}$ at an angle of θ above the horizontal from a point O on a horizontal plane and moves freely under gravity. The horizontal and vertical displacements of P from O at a subsequent time t s are denoted by x m and y m respectively.

(a) Starting from the equation of the trajectory given in the List of formulae (MF19), show that

$$y = x\tan\theta - \frac{gx^2}{2u^2}(1+\tan^2\theta).$$ [1]

When $\theta = \tan^{-1}2$, P passes through the point with coordinates (10, 16).

(b) Show that there is no value of θ for which P can pass through the point with coordinates (18, 30). [6]

6 One end of a light elastic string, of natural length a and modulus of elasticity k, is attached to a particle P of mass m. The other end of the string is attached to a fixed point Q. The particle P is projected vertically upwards from Q. When P is moving upwards and at a distance $\frac{4}{3}a$ directly above Q, it has a speed $\sqrt{2ga}$. At this point, its acceleration is $\frac{7}{3}g$ downwards.

Show that $k = 4mg$ and find in terms of a the greatest height above Q reached by P. [8]

7 A particle P of mass m kg moves in a horizontal straight line against a resistive force of magnitude mkv^2 N, where v m s^{-1} is the speed of P after it has moved a distance x m and k is a positive constant. The initial speed of P is u m s^{-1}.

(a) Show that $x = \frac{1}{k}\ln 2$ when $v = \frac{1}{2}u$. [4]

Beginning at the instant when the speed of P is $\frac{1}{2}u$, an additional force acts on P. This force has magnitude $\frac{5m}{v}$N and acts in the direction of increasing x.

(b) Show that when the speed of P has increased again to $u\,\mathrm{m\,s^{-1}}$, the total distance travelled by P is given by an expression of the form

$$\frac{1}{3k}\ln\left(\frac{A-ku^3}{B-ku^3}\right),$$

stating the values of the constants A and B. [7]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

BLANK PAGE

BLANK PAGE

Cambridge Assessment International Education

Cambridge International AS & A Level

FURTHER MATHEMATICS **9231/32**

Paper 3 Further Mechanics 32 **October/November 2020**

MARK SCHEME

Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2020 series for most Cambridge IGCSE™, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

[Turn over

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:
Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:
Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:
Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:
Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	At B, $mg\cos\beta = \frac{mv^2}{a}$: $(v^2 = ag\cos\beta)$	**B1**	
	$\frac{1}{2}mv^2 = \frac{1}{2}mu^2 + mga(\cos\alpha - \cos\beta)$	**M1A1**	Energy equation with 4 terms and correct dimensions $(v^2 = \frac{3ga}{2} - 2ag\cos\beta)$
	Substitute for u, $\cos\alpha$ and v: $ag\cos\beta = \frac{ag}{6} + 2ag\left(\frac{2}{3} - \cos\beta\right)$	**M1**	Eliminate to find $\cos\beta$
	$\cos\beta = \frac{1}{2}$	**A1**	
		5	

Question	Answer	Marks	Guidance
2	Speeds v and w after collision $2mv + mw = 2mu\cos\alpha$	**M1**	Momentum equation with m. Correct masses, allow sin instead of cos
	$w - v = eu\cos\alpha$	**M1**	Restitution, with consistent signs
	$v = \frac{1}{3}u\cos\alpha(2-e) = \frac{1}{3}u.\frac{3}{5}\left(2-\frac{1}{3}\right) = \frac{1}{3}u$	**A1**	
	Square of speed of $A = \left(\frac{1}{3}u\right)^2 + (u\sin\alpha)^2$	**M1**	Uses correct speed perpendicular to motion
	$= \left(\frac{1}{3}u\right)^2 + \left(\frac{4}{5}u\right)^2$ Speed $= \frac{13}{15}u$ $(= 0.867u)$	**A1**	
		5	

Question	Answer	Marks	Guidance
3(a)	<table><tr><th></th><th>Volume</th><th>Centre of mass from base</th></tr><tr><td>Cone</td><td>$\frac{1}{3}\pi(3r)^2.4r$</td><td>$4r+r$</td></tr><tr><td>Cylinder</td><td>$\pi(3r)^2.4r$</td><td>$2r$</td></tr><tr><td>Combined</td><td>$\frac{4}{3}\pi(3r)^2.4r$</td><td>$\bar{x}$</td></tr></table>	**B1**	Distances correct
	Taking moments about base of cylinder: $\bar{x}.\frac{4}{3}\pi(3r)^2.4r=\frac{1}{3}\pi(3r)^2.4r.5r+\pi(3r)^2.4r.2r$	**M1 A1**	Moments equation
	$\bar{x}=\frac{11}{4}r$	**A1**	
		4	
3(b)	Condition: $OG\cos\theta<OA$ (where O is vertex of cone and OA is slant height of cone)	**B1**	Correct condition for equilibrium
	$\left(4r+\frac{5r}{4}\right)\times\frac{4}{5}<5r$	**M1**	Expression in terms of r
	$21 < 25$ True	**A1**	Correct conclusion, with correct working
		3	

Question	Answer	Marks	Guidance
4(a)	$\uparrow N\cos\theta = mg$	**B1**	
	$\leftarrow N\sin\theta = mr\sin\theta\omega^2$	**B1**	
	$\cos\theta = \frac{mg}{N}$ so $\cos\theta = \frac{g}{\omega^2 r}$	**B1**	AG
		3	
4(b)	$\cos\theta = \frac{r-x}{r} = \frac{g}{\omega^2 r}$	**B1**	Using trig of situation: must involve x
	In new situation: $r - 4x = r \times \frac{g}{4\omega^2 r}$	**M1**	Using new situation with $4x$ and 2ω seen
	$r - x = 4(r - 4x)$	**M1**	Combining
	$x = \frac{1}{5}r$	**A1**	
		4	

Question	Answer	Marks	Guidance
5(a)	Quote trajectory equation from MF19 and use $\cos\theta = 1/\sec\theta$ $y = x\tan\theta - \frac{gx^2}{2u^2}(1+\quad \tan^2\theta)$	**B1**	Must include step with $\sec^2\theta$ Allow derived from first principles AG
		1	
5(b)	$16 = 20 - \frac{10\times100}{2u^2}(1+4)$	**M1**	Substitute into result **(a)**
	$u^2 = 625, \quad (u = 25)$	**A1**	
	Use equation again: $30 = 18\tan\theta - \frac{10\times324}{2\times625}\left(1+(\tan\theta)^2\right)$	**M1**	
	$2.592(\tan\theta)^2) - 18\tan\theta + 32.592 = 0$	**A1**	3 term quadratic. Alternatives include: $54t^2 - 375t + 679 = 0$, $324t^2 - 2250t + 4074 = 0$
	Discriminant = $324 - 4\times2.592\times32.592 = -13.91$	**M1**	Discriminant for alternatives: −6039 and −217404
	As this is less than 0, no real solutions for θ	**A1**	CWO
		6	

Question	Answer	Marks	Guidance
6	$T + mg = m.\frac{7}{3}g$	**M1**	
	With $T = k\frac{\frac{a}{3}}{a}$ giving $k = 4mg$	**A1**	AG
	Let greatest height above Q be $\frac{4}{3}a + x$ Gain in GPE = mgx and Loss in KE = ½ $m.2ga$	**B1**	The length being found may be expressed as the total extension of the string or the greatest height above Q. GPE and KE
	Gain in EPE = $\frac{1}{2}.\frac{4mg}{a}\left(\left(x+\frac{a}{3}\right)^2 - \left(\frac{a}{3}\right)^2\right)$	**B1**	EPE Note: initial EPE = $\frac{2mga}{9}$
	$\frac{4mg}{2a}\left(x^2 + \frac{2ax}{3} + \frac{a^2}{9} - \frac{a^2}{9}\right) + mgx = mga$	**M1 A1**	Energy equation, correct number of terms
	$2x^2 + \frac{7ax}{3} - a^2 = 0$	**M1**	Simplify to quadratic
	$x = \frac{1}{3}a$ so greatest height is $\frac{5}{3}a$	**A1**	

Question	Answer	Marks	Guidance
7(a)	$mv\frac{dv}{dx} = -kmv^2$	**B1**	N2L, with m
	$\ln v = -kx + c$	**M1**	Separate variables and integrate
	$x = 0, v = u: \quad c = \ln u$	**M1**	Use initial condition
	$v = \frac{1}{2}u: \quad \ln\frac{1}{2} = -kx,$ $x = \frac{1}{k}\ln 2$	**A1**	AG
		4	
7(b)	$mv\frac{\mathrm{d}v}{\mathrm{d}x} = -mkv^2 + \frac{5m}{v}$	**B1**	N2L (allow missing m in this part)
	$\frac{v^2\mathrm{d}v}{5-kv^3} = \mathrm{d}x \; -\frac{1}{3k}\ln\left(5-kv^3\right) = x(+d)$	**M1A1**	Separate variables and integrate
	Using **(a)** $-\frac{1}{3k}\ln\left(5-kv^3\right) = x - \frac{1}{3k}\ln\left(5-ku^3\right) - \frac{1}{k}\ln 2$	**M1M1**	Use condition. M0 if $v = \frac{1}{2}u, x = 0$ used unless $\frac{1}{k}\ln 2$ is added on later Rearrange dependent on ln solution
	$x = \frac{1}{3k}\ln\left(\frac{40-ku^3}{5-ku^3}\right)$	**M1A1**	Use $v = u$
		7	

Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME									
CENTRE NUMBER						CANDIDATE NUMBER			

FURTHER MATHEMATICS **9231/32**

Paper 3 Further Mechanics **May/June 2020**

1 hour 30 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.
- Where a numerical value for the acceleration due to gravity (g) is needed, use $10\,\text{m}\,\text{s}^{-2}$.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

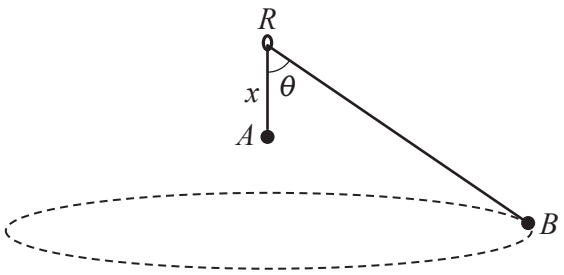
This document has **16** pages. Blank pages are indicated.

BLANK PAGE

1 A particle P is projected with speed u at an angle of 30° above the horizontal from a point O on a horizontal plane and moves freely under gravity. The particle reaches its greatest height at time T after projection.

Find, in terms of u, the speed of P at time $\frac{2}{3}T$ after projection. [5]

2



A light inextensible string of length a is threaded through a fixed smooth ring R. One end of the string is attached to a particle A of mass $3m$. The other end of the string is attached to a particle B of mass m. The particle A hangs in equilibrium at a distance x vertically below the ring. The angle between AR and BR is θ (see diagram). The particle B moves in a horizontal circle with constant angular speed $2\sqrt{\frac{g}{a}}$.

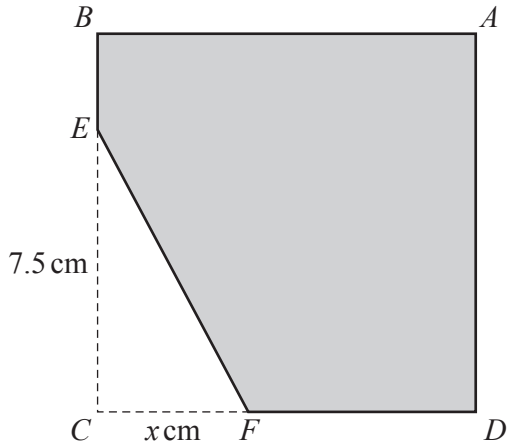
Show that $\cos\theta = \frac{1}{3}$ and find x in terms of a. [5]

3 One end of a light elastic spring, of natural length a and modulus of elasticity $5mg$, is attached to a fixed point A. The other end of the spring is attached to a particle P of mass m. The spring hangs with P vertically below A. The particle P is released from rest in the position where the extension of the spring is $\frac{1}{2}a$.

(a) Show that the initial acceleration of P is $\frac{3}{2}g$ upwards. [3]

(b) Find the speed of P when the spring first returns to its natural length. [4]

4



A uniform square lamina $ABCD$ has sides of length 10 cm. The point E is on BC with $EC = 7.5$ cm, and the point F is on DC with $CF = x$ cm. The triangle EFC is removed from $ABCD$ (see diagram). The centre of mass of the resulting shape $ABEFD$ is a distance $\bar{x}$ cm from CB and a distance $\bar{y}$ cm from CD.

(a) Show that $\bar{x} = \dfrac{400 - x^2}{80 - 3x}$ and find a corresponding expression for $\bar{y}$. [4]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

The shape *ABEFD* is in equilibrium in a vertical plane with the edge *DF* resting on a smooth horizontal surface.

(b) Find the greatest possible value of x, giving your answer in the form $a+b\sqrt{2}$, where a and b are constants to be determined. [3]

5 A particle P is moving along a straight line with acceleration $3ku - kv$ where v is its velocity at time t, u is its initial velocity and k is a constant. The velocity and acceleration of P are both in the direction of increasing displacement from the initial position.

(a) Find the time taken for P to achieve a velocity of $2u$. [3]

(b) Find an expression for the displacement of P from its initial position when its velocity is $2u$. [5]

6 A particle P of mass m is moving with speed u on a fixed smooth horizontal surface. The particle strikes a fixed vertical barrier. At the instant of impact the direction of motion of P makes an angle α with the barrier. The coefficient of restitution between P and the barrier is e. As a result of the impact, the direction of motion of P is turned through 90°.

(a) Show that $\tan^2\alpha = \frac{1}{e}$. [3]

The particle P loses two-thirds of its kinetic energy in the impact.

(b) Find the value of α and the value of e. [5]

7 A hollow cylinder of radius a is fixed with its axis horizontal. A particle P, of mass m, moves in part of a vertical circle of radius a and centre O on the smooth inner surface of the cylinder. The speed of P when it is at the lowest point A of its motion is $\sqrt{\frac{7}{2}ga}$.

The particle P loses contact with the surface of the cylinder when OP makes an angle θ with the upward vertical through O.

(a) Show that $\theta = 60°$. [5]

(b) Show that in its subsequent motion P strikes the cylinder at the point A. [5]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

Cambridge Assessment International Education

Cambridge International AS & A Level

FURTHER MATHEMATICS **9231/32**

Paper 3 Further Mechanics 3 **May/June 2020**

MARK SCHEME

Maximum Mark: 50

Published

Students did not sit exam papers in the June 2020 series due to the Covid-19 global pandemic.

This mark scheme is published to support teachers and students and should be read together with the question paper. It shows the requirements of the exam. The answer column of the mark scheme shows the proposed basis on which Examiners would award marks for this exam. Where appropriate, this column also provides the most likely acceptable alternative responses expected from students. Examiners usually review the mark scheme after they have seen student responses and update the mark scheme if appropriate. In the June series, Examiners were unable to consider the acceptability of alternative responses, as there were no student responses to consider.

Mark schemes should usually be read together with the Principal Examiner Report for Teachers. However, because students did not sit exam papers, there is no Principal Examiner Report for Teachers for the June 2020 series.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the June 2020 series for most Cambridge IGCSE™ and Cambridge International A & AS Level components, and some Cambridge O Level components.

[Turn over

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:
Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:
Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:
Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
Rules must be applied consistently e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:
Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks
1	For greatest height, $T = \frac{u}{2g}$	**B1**
	At $t = \frac{2T}{3}, \uparrow v_v = \frac{u}{2} - \frac{2Tg}{3} = \frac{u}{6}$	**M1**
	$\rightarrow v_h = \frac{u\sqrt{3}}{2}$	**A1**
	Speed $= \sqrt{v_v^2 + v_h^2} = \sqrt{\frac{u^2}{36} + \frac{3u^2}{4}}$	**M1**
	$= \frac{\sqrt{7}}{3}u$	**A1**
		5

Question	Answer	Marks
2	For A: $T = 3mg$ For B: $\uparrow T\cos\theta = mg$	**M1**
	Equate: $3mg\cos\theta = mg$ $\cos\theta = \frac{1}{3}$	**A1**
	$\rightarrow T\sin\theta = mr\omega^2$ with $r = (a-x)\sin\theta$	**M1**
	Equate: $3mg = m(a-x)\omega^2$	**A1**
	$x = \frac{a}{4}$	**A1**
		5

Question	Answer	Marks
3(a)	$T - mg = m.a$	**M1**
	$T = 5mg.\frac{1}{2}a / a = \frac{5}{2}mg$	**M1**
	$a = \frac{3}{2}g$ (upwards) **AG**	**A1**
		3

Question	Answer	Marks
3(b)	Gain in KE = $\frac{1}{2}mv^2$ Gain in GPE = $\frac{1}{2}mga$	**B1**
	Loss in EPE = $\frac{1}{2}\frac{5mg.\left(\frac{1}{2}a\right)^2}{a}$	**B1**
	$\frac{1}{2}mv^2+\frac{1}{2}mga=\frac{1}{2}\frac{5mg.\left(\frac{1}{2}a\right)^2}{a}$ [$\Rightarrow \frac{1}{2}mv^2+\frac{1}{2}mga=\frac{5}{8}mga$]	**M1**
	$v=\frac{1}{2}\sqrt{ga}$	**A1**
		4

Question	Answer	Marks
4(a)	<table><tr><th></th><th>Area</th><th>Centre of mass from BC</th><th>Centre of mass from DC</th></tr><tr><td>Square</td><td>100</td><td>5</td><td>5</td></tr><tr><td>Triangle</td><td>½ x. 15/2</td><td>$\frac{1}{3}x$</td><td>$\frac{5}{2}$</td></tr><tr><td>Shape $ABEFD$</td><td>$100-\frac{15}{4}x$</td><td>$\bar{x}$</td><td>$\bar{y}$</td></tr></table> Take moments about BC: $\left(100-\frac{15}{4}x\right)\sigma\,.\,\bar{x}=500\sigma-\frac{15}{4}x\sigma.\frac{1}{3}x$ (**M1** for all terms present)	**M1**
	$\bar{x}=\frac{400-x^2}{80-3x}$ **AG**	**A1**
	Take moments about DC: $\left(100-\frac{15}{4}x\right).\,\bar{y}\,=100\times5-\frac{15}{4}x.\frac{5}{2}$	**M1**
	$\bar{y}\,=\frac{800-15x}{160-6x}$	**A1**
		4
4(b)	Use condition: $\bar{x}\geqslant x$	**B1**
	$2x^2-80x+400\geqslant0$	**M1**
	$x=20-10\sqrt{2}$	**A1**
		3

Question	Answer	Marks
5(a)	$\frac{\mathrm{d}v}{3u-v}=k\mathrm{d}t$	**M1**
	$-\ln(3u-v)=kt+d$ $t=0, v=u:\quad d=-\ln 2u$	**M1**
	$v=2u:\quad t=\frac{1}{k}\ln 2$	**A1**
		3
5(b)	$v\frac{\mathrm{d}v}{\mathrm{d}x}=3ku-kv\ \left[\Rightarrow \frac{v\mathrm{d}v}{3u-v}=k\mathrm{d}x\right]$	**B1**
	$\frac{\left(-(3u-v)+3u\right)\mathrm{d}v}{3u-v}=k\mathrm{d}x$ so $-v-3u\ln(3u-v)=kx+c$	**M1A1**
	$x=0, v=u:\ c=-u-3u\ln 2u$	**M1**
	$v=2u:\ x=\frac{u}{k}(3\ln 2-1)$	**A1**
		5

Question	Answer	Marks
6(a)	Let components of velocity (parallel to plane and perpendicular) after impact be (x, y)	
	$y = v\cos\alpha = eu\sin\alpha$	**B1**
	$x = v\sin\alpha = u\cos\alpha$	**B1**
	Divide: $\tan\alpha = \dfrac{1}{e\tan\alpha} : \tan^2\alpha = \dfrac{1}{e}$.	**B1**
		3

Question	Answer	Marks
6(b)	$v^2 = \frac{1}{3}u^2$	**B1**
	$\left(\frac{u\cos\alpha}{\sin\alpha}\right)^2 = \frac{1}{3}u^2$	**M1**
	$(\tan\alpha)^2 = 3$	**M1**
	$\alpha = 60°$	**A1**
	$e = \frac{1}{3}$	**A1**
	Alternative method for **6(b)**	
	KE after impact $= \frac{1}{2}m\left(x^2 + y^2\right) = \frac{1}{2}m\left((u\cos\alpha)^2 + e^2(u\sin\alpha)^2\right)$	**M1**
	From **(a)** $\sin\alpha = 1/\sqrt{(1+e)}$ and $\cos\alpha = \sqrt{e}/\sqrt{(1+e)}$	**B1**
	KE $= \frac{1}{2}mu^2\left(\frac{e}{1+e} + \frac{e^2}{1+e}\right) = \frac{1}{2}mu^2 e$	**A1**
	This is equal to $\frac{1}{3}\times\frac{1}{2}mu^2$ so $e = \frac{1}{3}$	**M1**
	$\tan\alpha = \sqrt{3}$, $\alpha = 60°$	**A1**
		5

Question	Answer	Marks
7(a)	$(N+)mg\cos\theta = \frac{mv^2}{a}$	**B1**
	$\frac{1}{2}mv^2 - \frac{1}{2}m\frac{7ag}{2} = -mg(a + a\cos\theta)$	**M1A1**
	Loses contact when $N = 0$, so combine and simplify	**M1**
	$\cos\theta = \frac{1}{2}$: $\theta = 60°$ **AG**	**A1**
		5

Question	Answer	Marks
7(b)	When P is vertically below O, its horizontal displacement is $a\sin 60$, so time $T = \frac{a\sin 60}{v\cos 60} = a\sqrt{3}/v = \sqrt{\frac{6a}{g}}$	**M1**
	From **(a)**, $v^2 = \frac{1}{2}ag$	**A1**
	Vert: $h = \frac{v\sqrt{3}}{2}T - \frac{1}{2}g.T^2$	**M1**
	$\frac{3}{2}a - 3a = -\frac{3a}{2}$	**A1**
	This corresponds to the point A	**A1**
	Alternative method for question 7(b)	
	$y = x\sqrt{3} - \frac{4x^2}{a}$	**M1A1**
	Coordinates of A: $x = \frac{1}{2}a\sqrt{3}$, $y = -\frac{3}{2}a$	**B1**
	Substitute coordinates into $y = x\sqrt{3} - \frac{4x^2}{a}$ and show that these satisfy this equation	**M1A1**
		5

Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME									
CENTRE NUMBER						CANDIDATE NUMBER				

8623401314

FURTHER MATHEMATICS **9231/33**

Paper 3 Further Mechanics **May/June 2020**

1 hour 30 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.
- Where a numerical value for the acceleration due to gravity (g) is needed, use $10\,\text{m}\,\text{s}^{-2}$.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Blank pages are indicated.

1 A particle P of mass m is attached to one end of a light inextensible string of length a. The other end of the string is attached to a fixed point O on a smooth horizontal plane. The particle P moves in horizontal circles about O. The tension in the string is $4mg$.

Find, in terms of a and g, the time that P takes to make one complete revolution. [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

2 A particle Q of mass m kg falls from rest under gravity. The motion of Q is resisted by a force of magnitude mkv N, where v ms^{-1} is the speed of Q at time t s and k is a positive constant.

Find an expression for v in terms of g, k and t. [6]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

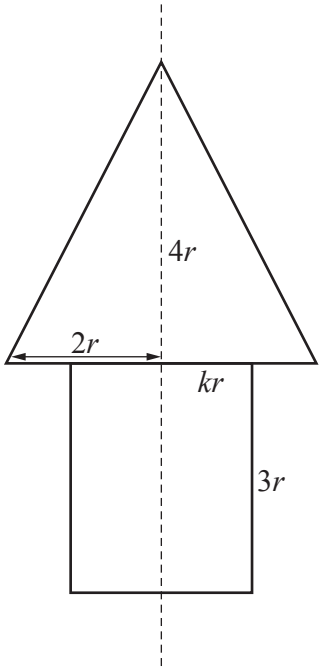
[Turn over

3 A particle Q of mass m is attached to a fixed point O by a light inextensible string of length a. The particle moves in complete vertical circles about O. The points A and B are on the path of Q with AB a diameter of the circle. OA makes an angle of 60° with the downward vertical through O and OB makes an angle of 60° with the upward vertical through O. The speed of Q when it is at A is $2\sqrt{ag}$.

Given that T_A and T_B are the tensions in the string at A and B respectively, find the ratio $T_A : T_B$. [6]

[Turn over

4



A uniform solid circular cone, of vertical height $4r$ and radius $2r$, is attached to a uniform solid cylinder, of height $3r$ and radius kr, where k is a constant less than 2. The base of the cone is joined to one of the circular faces of the cylinder so that the axes of symmetry of the two solids coincide (see diagram). The cone and the cylinder are made of the same material.

(a) Show that the distance of the centre of mass of the combined solid from the vertex of the cone is $\dfrac{(99k^2+96)r}{18k^2+32}$. [4]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

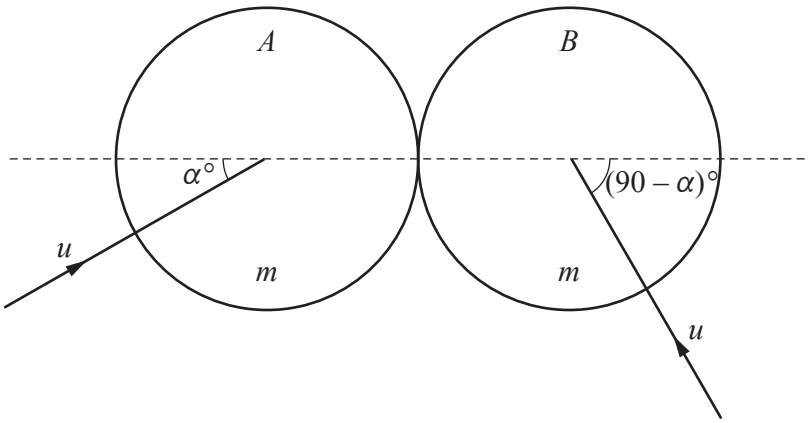
.....

.....

The point C is on the circumference of the base of the cone. When the combined solid is freely suspended from C and hanging in equilibrium, the diameter through C makes an angle α with the downward vertical, where $\tan\alpha = \frac{1}{8}$.

(b) Given that the centre of mass of the combined solid is within the cylinder, find the value of k. [4]

5



Two uniform smooth spheres A and B of equal radii each have mass m. The two spheres are each moving with speed u on a horizontal surface when they collide. Immediately before the collision A's direction of motion makes an angle of $\alpha°$ with the line of centres, and B's direction of motion is perpendicular to that of A (see diagram). The coefficient of restitution between the spheres is e.

Immediately after the collision, B moves in a direction at right angles to the line of centres.

(a) Show that $\tan\alpha = \dfrac{1+e}{1-e}$. [4]

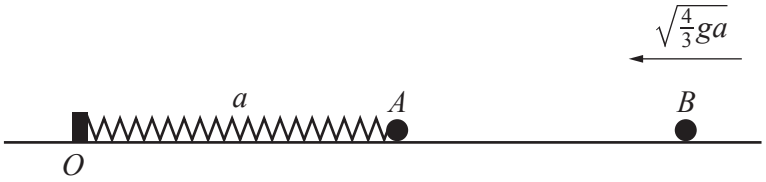
(b) Given that $\tan\alpha = 2$, find the speed of A after the collision. [4]

6 A particle P is projected with speed u at an angle θ above the horizontal from a point O on a horizontal plane and moves freely under gravity. The direction of motion of P makes an angle α above the horizontal when P first reaches three-quarters of its greatest height.

(a) Show that $\tan\alpha = \frac{1}{2}\tan\theta$. [6]

(b) Given that $\tan\theta = \frac{4}{3}$, find the horizontal distance travelled by P when it first reaches three-quarters of its greatest height. Give your answer in terms of u and g. [4]

7



One end of a light spring of natural length a and modulus of elasticity $4mg$ is attached to a fixed point O. The other end of the spring is attached to a particle A of mass km, where k is a constant. Initially the spring lies at rest on a smooth horizontal surface and has length a. A second particle B, of mass m, is moving towards A with speed $\sqrt{\frac{4}{3}ga}$ along the line of the spring from the opposite direction to O (see diagram).

The particles A and B collide and coalesce. At a point C in the subsequent motion, the length of the spring is $\frac{3}{4}a$ and the speed of the combined particle is half of its initial speed.

(a) Find the value of k. [6]

At the point C the horizontal surface becomes rough, with coefficient of friction μ between the combined particle and the surface. The deceleration of the combined particle at C is $\frac{9}{20}g$.

(b) Find the value of μ. [4]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

BLANK PAGE

BLANK PAGE

Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME									
CENTRE NUMBER						CANDIDATE NUMBER				

8623401314

FURTHER MATHEMATICS **9231/33**

Paper 3 Further Mechanics **May/June 2020**

1 hour 30 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.
- Where a numerical value for the acceleration due to gravity (g) is needed, use $10\,\text{m}\,\text{s}^{-2}$.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Blank pages are indicated.

1 A particle P of mass m is attached to one end of a light inextensible string of length a. The other end of the string is attached to a fixed point O on a smooth horizontal plane. The particle P moves in horizontal circles about O. The tension in the string is $4mg$.

Find, in terms of a and g, the time that P takes to make one complete revolution. [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

2 A particle Q of mass m kg falls from rest under gravity. The motion of Q is resisted by a force of magnitude mkv N, where v ms^{-1} is the speed of Q at time t s and k is a positive constant.

Find an expression for v in terms of g, k and t. [6]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

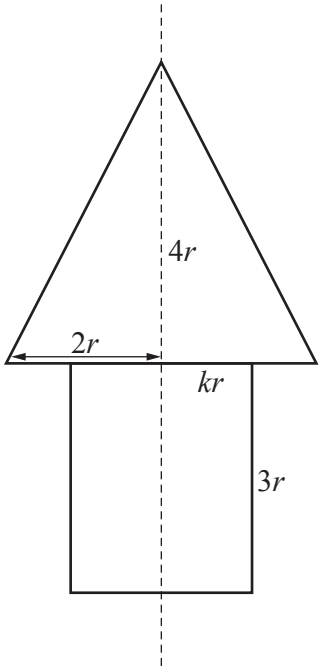
[Turn over

3 A particle Q of mass m is attached to a fixed point O by a light inextensible string of length a. The particle moves in complete vertical circles about O. The points A and B are on the path of Q with AB a diameter of the circle. OA makes an angle of 60° with the downward vertical through O and OB makes an angle of 60° with the upward vertical through O. The speed of Q when it is at A is $2\sqrt{ag}$.

Given that T_A and T_B are the tensions in the string at A and B respectively, find the ratio $T_A : T_B$. [6]

[Turn over

4



A uniform solid circular cone, of vertical height $4r$ and radius $2r$, is attached to a uniform solid cylinder, of height $3r$ and radius kr, where k is a constant less than 2. The base of the cone is joined to one of the circular faces of the cylinder so that the axes of symmetry of the two solids coincide (see diagram). The cone and the cylinder are made of the same material.

(a) Show that the distance of the centre of mass of the combined solid from the vertex of the cone is $\dfrac{(99k^2+96)r}{18k^2+32}$. [4]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

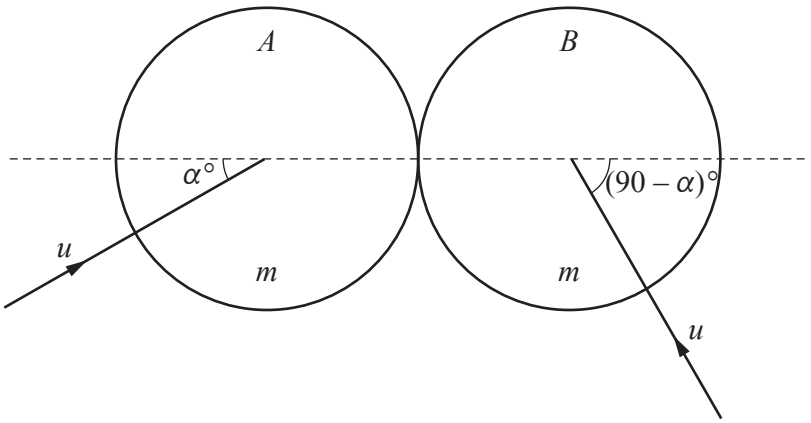
.....

.....

The point C is on the circumference of the base of the cone. When the combined solid is freely suspended from C and hanging in equilibrium, the diameter through C makes an angle α with the downward vertical, where $\tan\alpha = \frac{1}{8}$.

(b) Given that the centre of mass of the combined solid is within the cylinder, find the value of k. [4]

5



Two uniform smooth spheres A and B of equal radii each have mass m. The two spheres are each moving with speed u on a horizontal surface when they collide. Immediately before the collision A's direction of motion makes an angle of $\alpha°$ with the line of centres, and B's direction of motion is perpendicular to that of A (see diagram). The coefficient of restitution between the spheres is e.

Immediately after the collision, B moves in a direction at right angles to the line of centres.

(a) Show that $\tan\alpha = \dfrac{1+e}{1-e}$. [4]

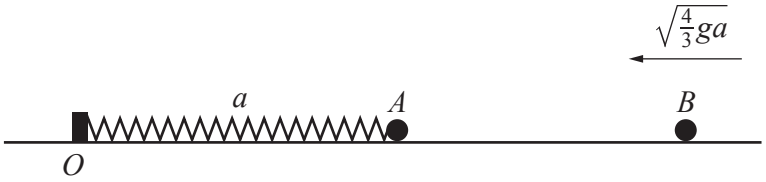
(b) Given that $\tan\alpha = 2$, find the speed of A after the collision. [4]

6 A particle P is projected with speed u at an angle θ above the horizontal from a point O on a horizontal plane and moves freely under gravity. The direction of motion of P makes an angle α above the horizontal when P first reaches three-quarters of its greatest height.

(a) Show that $\tan\alpha = \frac{1}{2}\tan\theta$. [6]

(b) Given that $\tan\theta = \frac{4}{3}$, find the horizontal distance travelled by P when it first reaches three-quarters of its greatest height. Give your answer in terms of u and g. [4]

7



One end of a light spring of natural length a and modulus of elasticity $4mg$ is attached to a fixed point O. The other end of the spring is attached to a particle A of mass km, where k is a constant. Initially the spring lies at rest on a smooth horizontal surface and has length a. A second particle B, of mass m, is moving towards A with speed $\sqrt{\frac{4}{3}ga}$ along the line of the spring from the opposite direction to O (see diagram).

The particles A and B collide and coalesce. At a point C in the subsequent motion, the length of the spring is $\frac{3}{4}a$ and the speed of the combined particle is half of its initial speed.

(a) Find the value of k. [6]

At the point C the horizontal surface becomes rough, with coefficient of friction μ between the combined particle and the surface. The deceleration of the combined particle at C is $\frac{9}{20}g$.

(b) Find the value of μ. [4]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

BLANK PAGE

BLANK PAGE