* 0000800000001 *

Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

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FURTHER MATHEMATICS **9231/31**

Paper 3 Further Mechanics **October/November 2024**

1 hour 30 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
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- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.
- Where a numerical value for the acceleration due to gravity (g) is needed, use $10\,\mathrm{m\,s^{-2}}$.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Any blank pages are indicated.

DC (DE) 336456/3

1 A particle P is projected with speed $u\,\text{m s}^{-1}$ at an angle $\tan^{-1}2$ above the horizontal from a point O on a horizontal plane and moves freely under gravity. When P has travelled a distance 56 m horizontally from O, it is at a vertical height H m above the plane. When P has travelled a distance 84 m horizontally from O, it is at a vertical height $\frac{1}{2}H$ m above the plane.

Find, in either order, the value of u and the value of H. [5]

2 A particle P of mass m is attached to one end of a light inextensible string of length a. The other end of the string is attached to a fixed point O. The particle P is held at the point A with the string taut. It is given that OA makes an angle θ with the downward vertical through O, where $\tan\theta = \frac{3}{4}$. The particle P is projected perpendicular to OA in an upwards direction with speed $\sqrt{5ag}$, and it starts to move along a circular path in a vertical plane. When P is at the point B, where angle AOB is a right angle, the tension in the string is T.

Find T in terms of m and g. [5]

3 A particle P of mass m kg is attached to one end of a light elastic string of natural length 2 m and modulus of elasticity $2mg$ N. The other end of the string is attached to a fixed point O. The particle P hangs in equilibrium vertically below O. The particle P is pulled down vertically a distance d m below its equilibrium position and released from rest.

(a) Given that the particle just reaches O in the subsequent motion, find the value of d. [6]

(b) Hence find the speed of P when it is 2 m below O. [2]

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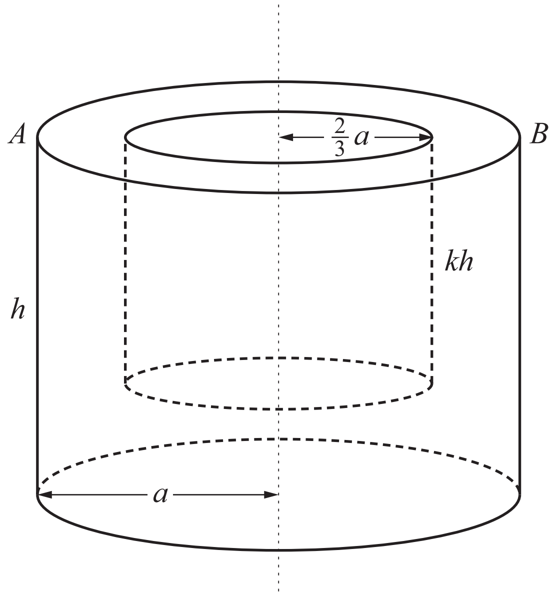

An object is formed by removing a cylinder of radius $\frac{2}{3}a$ and height kh ($k < 1$) from a uniform solid cylinder of radius a and height h. The vertical axes of symmetry of the two cylinders coincide. The upper faces of the two cylinders are in the same plane as each other. The points A and B are the opposite ends of a diameter of the upper face of the object (see diagram).

(a) Find, in terms of h and k, the distance of the centre of mass of the object from AB. [4]

When the object is suspended from A, the angle between AB and the vertical is θ, where $\tan\theta = \frac{3}{2}$.

(b) Given that $h = \frac{8}{3}a$, find the possible values of k. [3]

5 A particle P of mass 2 kg moving on a horizontal straight line has displacement x m from a fixed point O on the line and velocity $v\ \mathrm{m\,s^{-1}}$ at time t s. The only horizontal force acting on P is a variable force F N which can be expressed as a function of t. It is given that

$$\frac{v}{x} = \frac{3-t}{1+t}$$

and when $t = 0$, $x = 5$.

(a) Find an expression for x in terms of t. [4]

(b) Find the magnitude of F when $t = 3$. [3]

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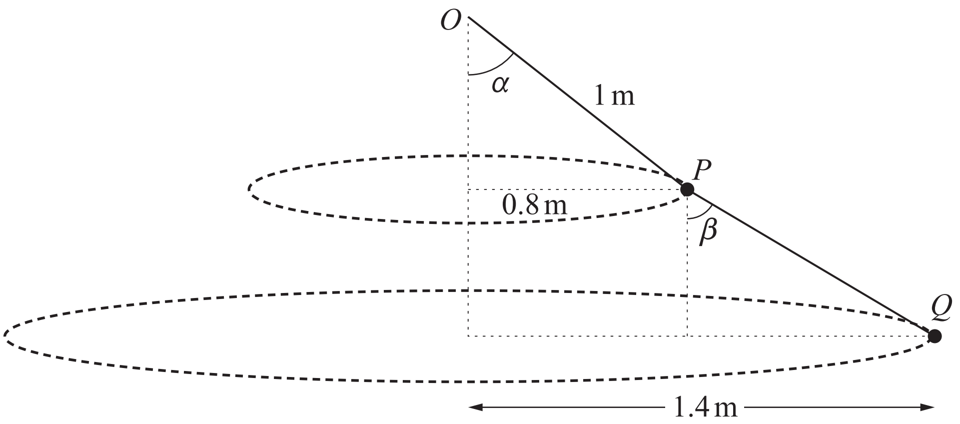

A particle P of mass 0.05 kg is attached to one end of a light inextensible string of length 1 m. The other end of the string is attached to a fixed point O. A particle Q of mass 0.04 kg is attached to one end of a second light inextensible string. The other end of this string is attached to P.

The particle P moves in a horizontal circle of radius 0.8 m with angular speed ω rad s^{-1}. The particle Q moves in a horizontal circle of radius 1.4 m also with angular speed ω rad s^{-1}. The centres of the circles are vertically below O, and O, P and Q are always in the same vertical plane. The strings OP and PQ remain at constant angles α and β respectively to the vertical (see diagram).

(a) Find the tension in the string OP. [3]

(b) Find the value of ω. [3]

(c) Find the value of β. [2]

7 A particle P is projected with speed u at an angle $\tan^{-1}\left(\frac{4}{3}\right)$ above the horizontal from a point O on a horizontal plane and moves freely under gravity. When P is moving horizontally, it strikes a smooth inclined plane at the point A. This plane is inclined to the horizontal at an angle α, and the line of greatest slope through A lies in the vertical plane through O and A.

As a result of the impact, P moves vertically upwards. The coefficient of restitution between P and the inclined plane is e.

(a) Show that $e\tan^2\alpha = 1$. [4]

In its subsequent motion, the greatest height reached by P above A is $\frac{3}{16}$ of the vertical height of A above the horizontal plane.

(b) Find the value of e. [6]

Additional page

If you use the following page to complete the answer to any question, the question number must be clearly shown.

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FURTHER MATHEMATICS **9231/31**

Paper 3 Further Mechanics **October/November 2024**

MARK SCHEME

Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

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Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2024 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptions for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:
Marks must be awarded in line with:
- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:
Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:
Marks must be awarded **positively**:
- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:
Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles

1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.

2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.

3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.

4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).

5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.

6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate’s intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	Use equation of trajectory with point (56, H) or $\left(84, \frac{1}{2}H\right)$	M1	For one equation with one error.
	$H = 112 - \frac{5g}{2u^2} \times 56^2$ or $\frac{1}{2}H = 168 - \frac{5g}{2u^2} \times 84^2$	A1	Both correct.
	Eliminate to find u or H	M1	
	$u = 35$	A1	
	$H = 48$	A1	
		5	

Question	Answer	Marks	Guidance
2	At B, $\mathrm{T} + mg\sin\theta = \frac{mv^2}{a}$	B1	
	Energy A to B: $\frac{1}{2}mu^2 - \frac{1}{2}mv^2 = mga(\cos\theta + \sin\theta)$	M1A1	
	Substitute for u and θ to find T: $T = mg\left(5 - 2 \times \frac{4}{5} - 3 \times \frac{3}{5}\right)$	M1	
	$T = \frac{8}{5}mg$	A1	
		5	

Question	Answer	Marks	Guidance
3(a)	Hooke's law: $T = \frac{2mg}{2} \times \text{extension}$ and $T = mg$	M1	Equilibrium position.
	Extension = 1 m	A1	
	EPE loss = $\frac{1}{2} \times \frac{2mg}{2} \times (1+d)^2$	B1	
	Gain in GPE = $mg(2+1+d)$	B1	
	Equate: $\frac{1}{2}mg(1+d)^2 = mg(3+d)$	M1	
	$d = \sqrt{5}$	A1	**SC**: 3 marks for final answer of $\sqrt{5}+1$. **SC**: 2 marks for final answer of $\sqrt{5}+k$, $k \neq 1$.
		6	
3(b)	Energy equation: $\frac{1}{2}mV^2 + mg(1+d) = \frac{1}{2} \times \frac{2mg}{2} \times (1+d)^2$	M1	GPE, KE, EPE terms.
	$V^2 = g(d^2-1)$ $V = \sqrt{40} = 2\sqrt{10}$	A1	
	Alternatively: Using KE and GPE from 2 m below O to point O $\frac{1}{2}mV^2 = 2mg$	M1	
	$V^2 = 4g$ $V = \sqrt{40} = 2\sqrt{10}$	A1	
		2	

<table>
<tr><th>Question</th><th>Answer</th><th>Marks</th><th>Guidance</th></tr>
<tr><td rowspan="3">4(a)</td><td rowspan="3">
<table>
<tr><td></td><td>Large</td><td>Small</td><td>Object</td></tr>
<tr><td>Volume</td><td>$\pi a^2 h$</td><td>$\pi\left(\frac{2}{3}a\right)^2 kh$</td><td>$\pi a^2 h\left(1-\frac{4}{9}k\right)$</td></tr>
<tr><td>Centre of mass from AB</td><td>$\frac{1}{2}h$</td><td>$\frac{1}{2}kh$</td><td>$\bar{x}$</td></tr>
</table>
Moments about AB:
$\pi a^2 h\left(1-\frac{4}{9}k\right)\times\bar{y} = \pi a^2 h\times\frac{1}{2}h-\pi\left(\frac{2}{3}a\right)^2 kh\times\frac{1}{2}kh$
</td><td>B1</td><td>Correct volumes and distances for large and small.</td></tr>
<tr><td>M1</td><td>Moments equation with 3 terms, dimensionally correct.</td></tr>
<tr><td>A1</td><td>Correct, unsimplified.</td></tr>
<tr><td></td><td>$\bar{y}=\frac{\left(9-4k^2\right)h}{2(9-4k)}$</td><td>A1</td><td></td></tr>
<tr><td></td><td></td><td>4</td><td></td></tr>
<tr><td>4(b)</td><td>$\tan\theta=\frac{\bar{y}}{a}\ :\ \frac{\left(9-4k^2\right)h}{2(9-4k)a}=\frac{3}{2}$</td><td>B1 FT</td><td>FT their part (a)</td></tr>
<tr><td></td><td>Use $h=\frac{8}{3}a$ and simplify to quadratic in k: $32k^2-36k+9=0$</td><td>M1</td><td></td></tr>
<tr><td></td><td>$k=\frac{3}{8},\quad \frac{3}{4}$</td><td>A1</td><td></td></tr>
<tr><td></td><td></td><td>3</td><td></td></tr>
</table>

Question	Answer	Marks	Guidance
5(a)	$\frac{dx}{x} = \left(\frac{4}{t+1} - 1\right)dt$	M1	Separate variables, obtain RHS in integrable form.
	$\ln\lvert x\rvert = 4\ln\lvert t+1\rvert - t + A$	A1	
	$t=0, x=5:\ A = \ln 5$	M1	
	$x = 5(t+1)^4 e^{-t}$	A1	
		4	
5(b)	$v = (3-t)\times 5(t+1)^3 e^{-t}$ Acceleration = $\frac{dv}{dt} = 5e^{-t}\left(-(t+1)^3 + (3-t)3(t+1)^2 - (3-t)(t+1)^3\right)$	M1	
	Acceleration = $5e^{-t}(t+1)^2(5-t)(1-t)$		AEF
	$F = 2\times$acceleration, so at $F = 10e^{-t}(t+1)^2(5-t)(1-t)$	M1	
	At $t=3$, magnitude of force is $640\,e^{-3}$ N	A1	31.9 N
		3	

Question	Answer	Marks	Guidance
6(a)	At P: $\uparrow$ $T_1 \cos\alpha = T_2 \cos\beta + 0.05g$	B1	
	At Q: $\uparrow$ $T_2 \cos\beta = 0.04g$	B1	OR: whole system: $T_1 \cos\alpha = 0.09g$
	$T_1 = 0.15g = 1.5$ N	B1	
		3	
6(b)	$T_1 \sin\alpha - T_2 \sin\beta = 0.05 \times 0.8\omega^2$	M1	Allow sin/cos mix
	$T_2 \sin\beta = 0.04 \times 1.4\omega^2$	M1	
	$T_1 \sin\alpha = 0.05 \times 0.8\omega^2 + 0.04 \times 1.4\omega^2$ $\omega^2 = 12.5, \quad \omega = \frac{5}{2}\sqrt{2}$	A1	
		3	
6(c)	$T_2 \cos\beta = 0.04g$ and $T_2 \sin\beta = 0.04 \times 1.4\omega^2$ Divide: $\tan\beta = \frac{7}{4}$	M1	From part **(a)** and part **(b)**
	$\beta = 60.3°$	A1	
		2	

Question	Answer	Marks	Guidance
7(a)	When P strikes plane, velocity is $\rightarrow u\cos\theta$, Before impact: parallel to inclined plane $\frac{3}{5}u\cos\alpha$, perpendicular to plane $\frac{3}{5}u\sin\alpha$	M1	$\frac{3}{5}u$
	After impact: components $\frac{3}{5}u\cos\alpha$ (parallel) and $\frac{3}{5}eu\sin\alpha$ (perpendicular)	A1	
	Since velocity is vertical after impact, $\tan\alpha = \frac{3}{5}u\cos\alpha \,/\, \frac{3}{5}eu\sin\alpha$	M1	
	$\tan\alpha = 1/e\tan\alpha$, $e\tan^2\alpha = 1$	A1	AG
		4	

Question	Answer	Marks	Guidance
7(b)	Greatest height of P before impact: $H=\dfrac{(u\sin\theta)^2}{2g}=\dfrac{8u^2}{25g}$	M1A1	Note: alternative methods.
	After impact, vertical speed of P is $\dfrac{3}{5}u\sqrt{(\cos\alpha)^2+e^2(\sin\alpha)^2}$	M1	
	Use $V^2=U^2+2as$ to greatest height, equal to $\dfrac{3}{16}H$ $\dfrac{9}{25}u^2\left((\cos\alpha)^2+e^2(\sin\alpha)^2\right)=2g\times\dfrac{3}{16}H$	M1	
	Use part **(a)**: $\tan\alpha=\dfrac{1}{\sqrt{e}}$, $\cos\alpha=\sqrt{\dfrac{e}{1+e}}$, $\sin\alpha=\sqrt{\dfrac{1}{1+e}}$ Substitute to find e	M1	
	$3e^2+2e-1=0,\quad e=\dfrac{1}{3}$	A1	
		6	

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DC (DE/CB) 337154/2

1 A particle of mass 2 kg is attached to one end of a light elastic string of natural length 0.8 m and modulus of elasticity 100 N. The other end of the string is attached to a fixed point O on a smooth horizontal surface. The particle is moving in a horizontal circle about O with the string taut and with constant angular speed 5 radians per second.

Find the extension of the string. [3]

DO NOT WRITE IN THIS MARGIN

2 A particle P of mass m is attached to one end of a light elastic spring of natural length a and modulus of elasticity $5mg$. The other end of the spring is attached to a fixed point O. The spring hangs vertically with P below O. The particle P is pulled down vertically and released from rest when the length of the spring is $\frac{3}{2}a$.

Find the distance of P below O when P first comes to instantaneous rest. [4]

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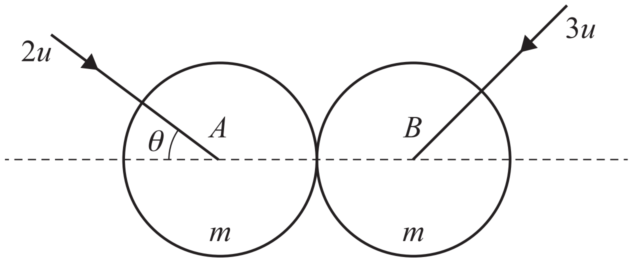

The diagram shows two identical smooth uniform spheres A and B of equal radii and each of mass m. The two spheres are moving on a smooth horizontal surface when they collide with speeds $2u$ and $3u$ respectively. Immediately before the collision, A's direction of motion makes an angle θ with the line of centres and B's direction of motion is perpendicular to that of A. After the collision, B moves perpendicular to the line of centres. The coefficient of restitution between the spheres is $\frac{1}{3}$.

(a) Find the value of $\tan\theta$. [3]

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(b) Find the total loss of kinetic energy as a result of the collision. [2]

(c) Find, in degrees, the angle through which the direction of motion of A is deflected as a result of the collision. [2]

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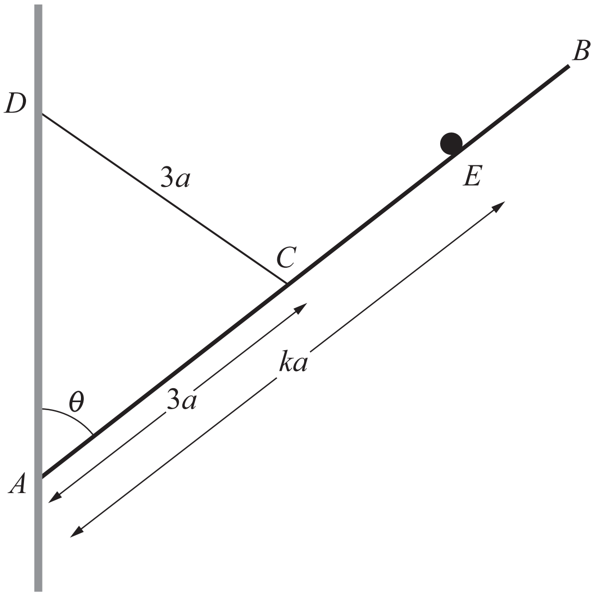

The end A of a uniform rod AB of length $6a$ and weight W is in contact with a rough vertical wall. One end of a light inextensible string of length $3a$ is attached to the midpoint C of the rod. The other end of the string is attached to a point D on the wall, vertically above A. The rod is in equilibrium when the angle between the rod and the wall is θ, where $\tan\theta = \frac{3}{2}$. A particle of weight W is attached to the point E on the rod, where the distance AE is equal to ka $(3 < k < 6)$ (see diagram). The rod and the string are in a vertical plane perpendicular to the wall. The coefficient of friction between the rod and the wall is $\frac{1}{3}$. The rod is about to slip down the wall.

(a) Find the value of k. [5]

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(b) Find, in terms of W, the magnitude of the frictional force between the rod and the wall. [2]

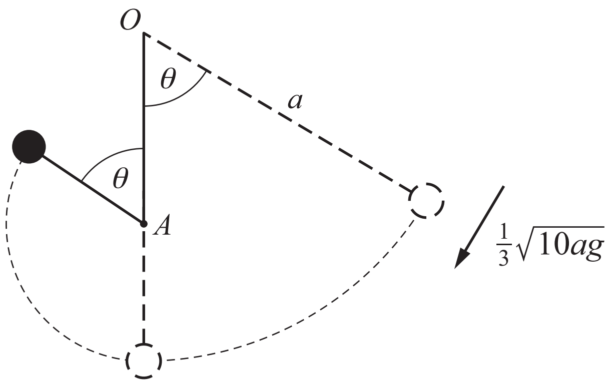
5 A particle P is projected from a point O on horizontal ground with speed u at an angle θ above the horizontal, where $\tan\theta = \frac{1}{3}$. The particle P moves freely under gravity and passes through the point with coordinates $(3a, \frac{4}{5}a)$ relative to horizontal and vertical axes through O in the plane of the motion.

(a) Use the equation of the trajectory to show that $u^2 = 25ag$. [2]

At the instant when P is moving horizontally, a particle Q is projected from O with speed V at an angle α above the horizontal. The particles P and Q reach the ground at the same point and at the same time.

(b) Express V^2 in the form kag, where k is a rational number. [6]

6



A particle P of mass m is attached to one end of a light inextensible string of length a. The other end of the string is attached to a fixed point O. The particle P is held with the string taut and the string makes an angle θ with the downward vertical through O. The particle P is projected at right angles to the string with speed $\frac{1}{3}\sqrt{10ag}$ and begins to move downwards along a circular path. When the string is vertical, it strikes a small smooth peg at the point A which is vertically below O. The circular path and the point A are in the same vertical plane. After the string strikes the peg, the particle P begins to move in a vertical circle with centre A. When the string makes an angle θ with the upward vertical through A the string becomes slack (see diagram). The distance of A below O is $\frac{5}{9}a$.

(a) Find the value of $\cos\theta$. [6]

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(b) Find the ratio of the tensions in the string immediately before and immediately after it strikes the peg. [4]

7 A particle P of mass m kg is held at rest at a point O and released so that it moves vertically under gravity against a resistive force of magnitude $0.1mv^2$ N, where v m s^{-1} is the velocity of P at time t s.

(a) Find an expression for v in terms of t. [6]

The displacement of P from O at time t s is x m.

(b) Find an expression for v^2 in terms of x. [5]

Additional page

If you use the following page to complete the answer to any question, the question number must be clearly shown.

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Cambridge International is publishing the mark schemes for the October/November 2024 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptions for a question. Each question paper and mark scheme will also comply with these marking principles.

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GENERIC MARKING PRINCIPLE 6: Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles

1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.

2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.

3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.

4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).

5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.

6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	Hooke's law: $T = \frac{100}{0.8}x$	B1	$T = \frac{100}{0.8}(r - 0.8)$
	N2L: $T = 2 \times (0.8 + x) \times 5^2$	B1	$T = 2 \times r \times 5^2$
	Equate and solve: $50(0.8 + x) = 125x$, $x = 0.533$	B1	$\frac{8}{15}$
		3	

Question	Answer	Marks	Guidance
2	Extension when P comes to rest is x, EPE loss = $\frac{1}{2}\times\frac{5mg}{a}\times\left(\left(\frac{a}{2}\right)^2 - x^2\right)$	B1	Both terms seen.
	Energy: $\frac{1}{2}\times\frac{5mg}{a}\times\left(\left(\frac{a}{2}\right)^2 - x^2\right) = mg\left(\frac{a}{2} - x\right)$	M1	At least one EPE term and a GPE term, dimensionally correct.
	Solve: $20x^2 - 8ax - a^2 = 0$	M1	Obtain homogeneous quadratic equation in x and a Must come from an energy equation involving two EPE terms. Note that the correct case simplifies to a linear equation $\frac{5mg}{2a}\left(\frac{a}{2} + x\right) = mg$ and this scores M1.
	$x = -\frac{a}{10}$, so distance of P below O is $\frac{9}{10}a$	A1	

Question	Answer	Marks	Guidance
2	**Alternative method for question 2**		
	Distance of P below O when it comes to rest comes to rest is h EPE loss = = $\frac{1}{2}\times\frac{5mg}{a}\times\left(\left(\frac{a}{2}\right)^2-(h-a)^2\right)$	**B1**	
	Energy: $\frac{1}{2}\times\frac{5mg}{a}\times\left(\left(\frac{a}{2}\right)^2-(h-a)^2\right)=mg\left(\frac{3}{2}a-h\right)$	**M1**	At least one EPE term and a GPE term, dimensionally correct.
	$20h^2-48ah+27a^2=0$	**M1**	Obtain homogeneous quadratic/linear equation in h and a. Must come from an energy equation involving two EPE terms.
	$(10h-9a)(2h-3a)=0,\quad h=\frac{9}{10}a$	**A1**	
		4	

Question	Answer	Marks	Guidance
3(a)	PCLM along line of centres: $-mv = m2u\cos\theta - m3u\sin\theta$	**B1**	Must include *m*, must have minus sign on RHS, accept positive or negative *v*. If velocity of *B* after collision is included, it must be equated to zero before this mark is awarded.
	NEL: $v = eu(3\sin\theta + 2\cos\theta)$	**M1**	Must have plus sign on RHS, accept positive or negative *v* (sign of *v* does **not** need to be consistent with PCLM equation) If velocity of *B* after collision is included, it must be equated to zero before this mark is awarded.
	Eliminate *v*: $6\sin\theta = 8\cos\theta, \quad \tan\theta = \frac{4}{3}$	**A1**	Correct work only, except possibly missing *m*.
		3	

Question	Answer	Marks	Guidance
3(b)	Only change in KE is along line of centres Loss = $\frac{1}{2}m\left((2u\cos\theta)^2+(3u\sin\theta)^2\right)-\frac{1}{2}mv^2$	M1	
	$\frac{1}{2}mu^2\left(\frac{36}{25}+\frac{144}{25}-\left(\frac{12}{5}-\frac{6}{5}\right)^2\right)=\frac{72}{25}mu^2$	A1	(Note that $\tan\theta=\frac{2}{3}$ leads to final answer $\frac{36}{13}mu^2$)
		2	
	Alternative method for question 3(b)		
	Alternative method, using total KE Loss in KE = [$\frac{1}{2}m(2u)^2+\frac{1}{2}m(3u)^2$] $-\left[\frac{1}{2}mv^2+\frac{1}{2}m(2u\sin\theta)^2+\frac{1}{2}m(3u\cos\theta)^2\right]$	M1	Or equivalent, with all necessary terms present
	$\frac{13}{2}mu^2-\frac{181}{50}mu^2=\frac{72}{25}mu^2$	A1	
		2	
3(c)	[Components of velocity of A after collision are $\leftarrow\frac{6}{5}u$ $\downarrow\frac{8}{5}u$ so] angle between line of centres and A's direction is θ.	M1	
	Angle of deflection = 180° – 2tan^{-1} 4/3 = 73.7°	A1FT	FT their answer to part **(a)**
		2	

Question	Answer	Marks	Guidance
4(a)	In this question, allow equivalent marks for resolutions in different directions and moments about other points. Apply the guidance given in the main scheme. $\uparrow T\cos\theta + F = 2W$	B1	
	$\rightarrow T\sin\theta = R$	B1	
	Moments about A: $T\cos\theta \times 3a\sin\theta + T\sin\theta \times 3a\cos\theta = W \times 3a\sin\theta + W \times ka\sin\theta$ OR Moments about C: $R \times 3a\cos\theta = F \times 3a\sin\theta + W \times (ka - 3a)\sin\theta$	M1	All relevant terms included, dimensionally correct, forces must be resolved if appropriate. Allow sin/cos mix, allow sign errors. LHS: any equivalent expression, for example $3Ta\sin 2\theta$, $3Ta\sin(180-2\theta)$. All relevant terms included, dimensionally correct, forces must be resolved if appropriate. Allow sin/cos mix, allow sign errors.
	[$6aT\cos\theta = (3+k)aW$ and $\frac{T}{3}(3\cos\theta + \sin\theta) = 2W$ give] [*and giveand give*] $12\cos\theta = (3+k)(\cos\theta + \frac{1}{3}\sin\theta)$	M1	Use $F = \frac{1}{3}R$ and eliminate T and W to obtain an expression in k and θ, dependent on a dimensionally correct moments equation.
	$k = 5$	A1	
		5	

Question	Answer	Marks	Guidance
4(b)	A complete method to find F in terms of W	M1	Any complete method to find F. For example, substitute into moments equation to obtain T [$T=\frac{2}{3}\sqrt{13}\ W,\ R=2W$] .
	$F=\frac{2}{3}W$	A1	Correct.
		2	

Question	Answer	Marks	Guidance
5(a)	Use correct equation of trajectory: $\frac{4}{5}a=3a\times\frac{1}{3}-\frac{g}{2u^2}\times(3a)^2\times\left(1+\frac{1}{9}\right)$	M1	No (implied) sight of trajectory equation M0.
	$\frac{4}{5}a=a-\frac{5ga^2}{u^2},\ \frac{5ga}{u^2}=\frac{1}{5},\ u^2=25ga$	A1	At least one step of intermediate working must be seen. AG
		2	

Question	Answer	Marks	Guidance
5(b)	For P, time of flight T and range R For Q, time of flight ½ T and range R $\left[T=\frac{2u\sin\theta}{g}=\right]\sqrt{\frac{10a}{g}}$	B1	Time of flight for P or Q.
	[From motion of P, $R=\frac{2}{g}\times 25ag\times\frac{3}{10}=$] $15a$	B1	Range for P.
	For Q: $\rightarrow R=v\cos\alpha\times\frac{1}{2}T$, $v\cos\alpha=\frac{2R}{T}$	M1	Obtain an expression for $v\cos\alpha$. May involve u and θ.
	$\uparrow$ $0=v\sin\alpha\times\frac{1}{2}T-\frac{1}{2}g\left(\frac{1}{2}T\right)^2$, $v\sin\alpha=\frac{1}{4}gT$	M1	Obtain an expression for $v\sin\alpha$. May involve u and θ.
	Square and add: $v^2=\left(\frac{2R}{T}\right)^2+\left(\frac{1}{4}gT\right)^2$ $\left[=90ag+\frac{5}{8}ag\right]$	M1	
	$v^2=\frac{725}{8}ag$	A1	$\tan\alpha=\frac{\frac{1}{4}gT}{\frac{2R}{T}}=\frac{1}{12}$

Question	Answer	Marks	Guidance
5(b)	**Alternative method for question 5(b)**		
	For P, time of flight T and range R For Q, time of flight ½ T and range R Horizontal motion for P and Q $R = u\cos\theta T$ and $R = (v\cos\alpha)\dfrac{T}{2}$	**M1**	Both.
	Vertical motion for P and Q $u\sin\theta = \dfrac{gT}{2}$ and $v\sin\alpha = \dfrac{gT}{4}$	**M1**	Both, may come from using $s = ut + \dfrac{1}{2}at^2$.
	Equate two expressions for R: $v\cos\alpha = 2u\cos\theta$	**A1**	$v\cos\alpha = \dfrac{6}{\sqrt{10}}u$
	Equate two expressions for vertical motion: $v\sin\alpha = \dfrac{1}{2}u\sin\theta$	**A1**	$v\sin\alpha = \dfrac{1}{2\sqrt{10}}u$
	Square and add: $v^2 = u^2\left(4\cos^2\theta + \dfrac{1}{4}\sin^2\theta\right)\left[= \dfrac{29}{8}u^2\right]$	**M1**	
	$\dfrac{29}{8} \times 25ag = \dfrac{725}{8}ag$	**A1**	
		6	

Question	Answer	Marks	Guidance
6(a)	For P to lowest point L: Energy: $\frac{1}{2}mv^2 = \frac{1}{2}mu^2 + mga(1-\cos\theta)$ $[v^2 = \frac{28}{9}ag - 2ag\cos\theta]$	M1*	Dimensionally correct, all terms present, allow sign errors, allow cos/sin error.
	From L to string goes slack: Energy: $\frac{1}{2}mv^2 = \frac{1}{2}mw^2 + \frac{4a}{9}mg(1+\cos\theta)$ $\left[w^2 = \frac{20}{9}ag - \frac{26}{9}ag\cos\theta\right]$	M1*	Dimensionally correct, all terms present, with $\frac{4a}{9}$, allow sign errors, allow cos/sin error.
	Both equations correct, allow unsimplified.	A1	
	When string goes slack: $mg\cos\theta = \frac{mw^2}{\frac{4a}{9}}$	B1	
	Equate expressions for w^2 to find a value for $\cos\theta$	DM1	
	$\cos\theta = \frac{2}{3}$	A1	
		6	

Question	Answer	Marks	Guidance
	Alternative method for question 6(a)		
	For P from start to string goes slack: Energy: $\frac{1}{2}mw^2 = \frac{1}{2}mu^2 + mga\left[(1-\cos\theta) - \frac{4}{9}(1+\cos\theta)\right]$	**M2**	Dimensionally correct, all terms present, with $\frac{4}{9}a$ Allow sign errors, allow cos/sin error RHS may appear with $\frac{5}{9}a - \left(1+\frac{4}{9}\right)a\cos\theta$ Allow M1 if $\frac{4}{9}$ is missing or if an attempt at both heights, but all other conditions are met.
	$\left[w^2 = \frac{10}{9}ag + 2ag\left(\frac{5}{9} - \frac{13}{9}\cos\theta\right) = \frac{20}{9}ag - \frac{26}{9}ag\cos\theta\right]$	**A1**	Correct, allow unsimplified.
	When string goes slack: $mg\cos\theta = \frac{mw^2}{\frac{4a}{9}}$	**B1**	
	Equate expressions for w^2 to find a value for $\cos\theta$	**DM1**	
	$\cos\theta = \frac{2}{3}$	**A1**	
		6	

Question	Answer	Marks	Guidance
6(b)	Tension before: $T_1 - mg = \dfrac{mv^2}{a}$ Tension after: $T_2 - mg = \dfrac{mv^2}{\frac{4}{9}a}$	M1	EITHER equation, dimensionally correct, allow sign error only
	$\left[v^2 = \dfrac{16}{9}ag,\right]$ $T_1 = \dfrac{25}{9}mg$ $T_2 = 5mg$	A1	EITHER tension correct
	Find the other tension (from a valid equation) and find ratio of tensions	M1	Equation must be of the form $T - mg = \dfrac{mv^2}{r}$
	Ratio is 5 : 9	A1	Any equivalent ratio, allow $\dfrac{5}{9}$
		4	

Question	Answer	Marks	Guidance
7(a)	$m\dfrac{dv}{dt} = mg - 0.1mv^2$	B1	Use of suvat means 0 marks in this part Note that no *mg* term means 0 marks in this part. Must see *m*, may be cancelled before $a = \dfrac{dv}{dt}$ used
	$\dfrac{dv}{dt} = 10 - 0.1v^2 = \dfrac{1}{10}\left(100 - v^2\right)$ $\dfrac{dv}{100 - v^2} = \dfrac{1}{10}dt$ $\ln\left\|\left(\dfrac{v+10}{10-v}\right)\right\| = 2t + A$	M1*	Separate variables and integrate. May see partial fractions, but integral is on Formula sheet, allow missing $+A$ for M1 only
		A1	Must see modulus sign
	Use $t = 0, v = 0,\ \ A = 0$	DM1	
	Remove logs to obtain *v* in terms of *t*	M1	
	$v = \dfrac{10\left(e^{2t} - 1\right)}{e^{2t} + 1}$ aef	A1	$v = \dfrac{10\left(1 - e^{-2t}\right)}{1 + e^{-2t}}$
		6	

Question	Answer	Marks	Guidance
7(b)	$v\dfrac{\mathrm{d}v}{\mathrm{d}x}=\dfrac{1}{10}\left(100-v^2\right)$	M1*	Use of suvat means 0 marks in this part Separate variables and integrate, allow missing $+A$ for M1 only.
	$-\dfrac{1}{2}\ln\left(\left\|100-v^2\right\|\right)=\dfrac{1}{10}x+B$	A1	For A1, allow missing modulus sign
	Use $x=0, v=0$, $B=-\dfrac{1}{2}\ln 100$	DM1	
	Remove logs to obtain v^2 in terms of x	M1	
	$v^2=100(1-\mathrm{e}^{-\frac{x}{5}})$	A1	AEF Allow $10g$ instead of 100.
		5	

Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

9063352144

FURTHER MATHEMATICS **9231/32**

Paper 3 Further Mechanics **May/June 2024**

1 hour 30 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.
- Where a numerical value for the acceleration due to gravity (g) is needed, use 10 m s^{-2}.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages.

DC (DE/CGW) 331450/1

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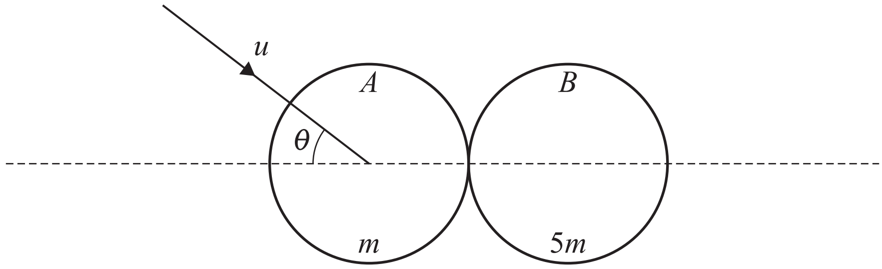

Two smooth uniform spheres A and B of equal radii have masses m and $5m$ respectively. Sphere A is moving on a smooth horizontal surface with speed u when it collides with sphere B which is at rest on the surface. Immediately before the collision, A's direction of motion makes an angle of θ with the line of centres. After the collision, the kinetic energies of A and B are equal. The coefficient of restitution between the spheres is $\frac{1}{2}$.

Find the value of $\tan\theta$. [6]

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2 The points A and B are at the same horizontal level a distance $4a$ apart. The ends of a light elastic string, of natural length $4a$ and modulus of elasticity λ, are attached to A and B. A particle P of mass m is attached to the midpoint of the string. The system is in equilibrium with P at a distance $\frac{3}{2}a$ below M, the midpoint of AB.

(a) Find λ in terms of m and g. [3]

The particle P is pulled down vertically and released from rest at a distance $\frac{8}{3}a$ below M.

(b) Find, in terms of a and g, the speed of P as it passes through M in the subsequent motion. [4]

3 At time $t = 0$ seconds, a particle P is projected with speed $u \text{ m s}^{-1}$ at an angle $60°$ above the horizontal from a point O. In the subsequent motion P moves freely under gravity. The direction of motion of P when $t = 5$ is perpendicular to its direction of motion when $t = 15$.

Find the value of u. [5]

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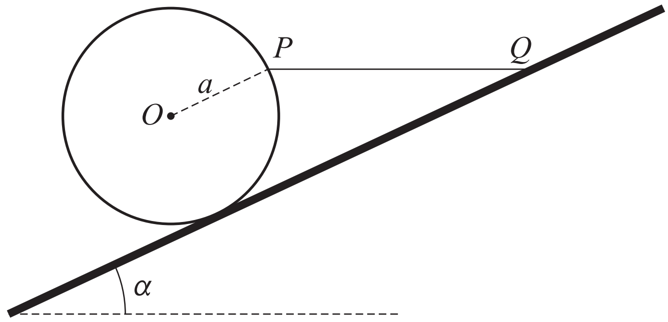

A ring of weight W, with radius a and centre O, is at rest on a rough surface that is inclined to the horizontal at an angle α where $\tan\alpha = \frac{1}{2}$. The plane of the ring is perpendicular to the inclined surface and parallel to a line of greatest slope of the surface. The point P on the circumference of the ring is such that OP is parallel to the surface.

A light inextensible string is attached to P and to the point Q, which is on the surface, such that PQ is horizontal (see diagram). The points O, P and Q are in the same vertical plane. The system is in limiting equilibrium and the coefficient of friction between the ring and the surface is μ.

(a) Find, in terms of W, the tension in the string PQ. [4]

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(b) Find the value of μ. [3]

5 Two particles A and B of masses m and km respectively are connected by a light inextensible string of length a. The particles are placed on a rough horizontal circular turntable with the string taut and lying along a radius of the turntable. Particle A is at a distance a from the centre of the turntable and particle B is at a distance $2a$ from the centre of the turntable. The coefficient of friction between each particle and the turntable is $\frac{1}{5}$.

When the turntable is made to rotate with angular speed $\frac{2}{5}\sqrt{\frac{g}{a}}$, the system is in limiting equilibrium.

(a) Find the tension in the string, in terms of m and g. [4]

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(b) Find the value of k. [3]

6 A particle P of mass 2 kg moving on a horizontal straight line has displacement x m from a fixed point O on the line and velocity $v\,\mathrm{m\,s^{-1}}$ at time t s. The only horizontal force acting on P has magnitude $\frac{1}{10}(2v-1)^2\mathrm{e}^{-t}$ N and acts towards O. When $t = 0$, $x = 1$ and $v = 3$.

(a) Find an expression for v in terms of t. [5]

(b) Find an expression for x in terms of t. [4]

7 A smooth sphere with centre O and of radius a is fixed to a horizontal plane. A particle P of mass m is projected horizontally from the highest point of the sphere with speed u, so that it begins to move along the surface of the sphere. The particle P loses contact with the sphere at the point Q on the sphere, where OQ makes an angle θ with the upward vertical through O.

(a) Show that $\cos\theta = \dfrac{u^2 + 2ag}{3ag}$. [4]

It is given that $\cos\theta = \frac{5}{6}$.

(b) Find, in terms of a and g, an expression for the vertical component of the velocity of P just before it hits the horizontal plane to which the sphere is fixed. [3]

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(c) Find an expression for the time taken by P to fall from Q to the plane. Give your answer in the form $k\sqrt{\frac{a}{g}}$, stating the value of k correct to 3 significant figures. [2]

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Additional page

If you use the following page to complete the answer to any question, the question number must be clearly shown.

Cambridge International AS & A Level

FURTHER MATHEMATICS **9231/32**

Paper 3 Further Mechanics **May/June 2024**

MARK SCHEME

Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

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GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles

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2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.

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Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

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DM or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
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- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
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SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	Along line of centres, PCLM: $5mv_B + mv_A = mu\cos\theta$	M1	Must include correct masses.
	NEL: $v_B - v_A = \frac{1}{2}u\cos\theta$	M1	Signs consistent with PCLM equation.
	$v_B = \frac{u}{4}\cos\theta,\ v_A = -\frac{u}{4}\cos\theta$	A1	
	Perpendicular to line of centres: speed of A is $u\sin\theta$	B1	
	$\frac{1}{2}m\left(\left(-\frac{u}{4}\cos\theta\right)^2 + (u\sin\theta)^2\right) = \frac{1}{2}5m\left(\frac{u}{4}\cos\theta\right)^2$	M1	Equate final kinetic energies, 3 terms, correct masses.
	$(\cos\theta)^2 = \frac{4}{5},\quad \cos\theta = \frac{2}{\sqrt{5}},\quad \tan\theta = \frac{1}{2}$	A1	
		6	

Question	Answer	Marks	Guidance
2(a)	In equilibrium: $2T\cos\theta = mg$	M1	
	Hooke's law: $T = \frac{\lambda}{2a} \times \left(\frac{5a}{2} - 2a\right) = \frac{\lambda}{4}$	B1	
	Equate and use $\cos\theta = \frac{3}{5}$: $\lambda = \frac{10}{3}mg$	A1	
		3	
2(b)	EPE loss = $\frac{1}{2}\frac{\lambda}{4a}\left(\frac{20}{3}a - 4a\right)^2$	B1	$\frac{80}{27}mga$
	Energy equation: $\frac{1}{2}mv^2 + mg\frac{8a}{3} = '\frac{80}{27}mga'$	M1 A1	All 3 terms required, dimensionally correct, their λ.
	$v = \frac{4}{3}\sqrt{\frac{ga}{3}}$	A1	Any equivalent form.
		4	

Question	Answer	Marks	Guidance
3	$u\sin 60° - 5g$, and $u\cos 60°$, or $u\sin 60° - 15g$, and $u\cos 60°$	**B1**	
	If θ is direction of velocity at $t = 5, [\tan\theta =]\dfrac{u\sin 60° - 5g}{u\cos 60°}$	**M1***	Accept equivalent for $t = 15$.
	For perpendicular directions, $\dfrac{u\sin 60° - 5g}{u\cos 60°} \times \dfrac{u\sin 60° - 15g}{u\cos 60°} = -1$	**M1dep**	Multiply two expressions involving relevant velocities and equate to – 1.
	Simplify: $\frac{3}{4}u^2 + 75g^2 - 10\sqrt{3}ug + \frac{1}{4}u^2 = 0, \quad u^2 - 100\sqrt{3}u + 7500 = 0$	**M1**	Simplify to quadratic in u (may see g).
	$u = 5\sqrt{3}g$	**A1**	OE. Accept $50\sqrt{3}$ or 86.6.
		5	

Question	Answer	Marks	Guidance
4(a)	Frictional force F and normal reaction R at point of contact of ring with plane. Resolve parallel to plane: $F + T\cos\alpha = W\sin\alpha$	**M1**	Only allow cos/sin errors for T and W components, sign errors. Accept equations for vertical and horizontal (both needed).
	Moments about O: $Fa = Ta\sin\alpha$	**B1**	
	Combine and substitute for α :	**M1**	Expression for T in terms of W .
	$T = \frac{1}{3}W$	**A1**	CAO
	Alternative solution for question 4(a)		
	Moments about point where ring touches plane: $Ta\sin\alpha + Ta\cos\alpha = Wa\sin\alpha$	**M1 A1**	Only allow cos/sin errors, sign errors. Must be dimensionally correct.
	Rearrange and substitute for α :	**M1**	Expression for T in terms of W .
	$T = \frac{1}{3}W$	**A1**	CAO
		4	
4(b)	Resolve perpendicular to plane: $R = T\sin\alpha + W\cos\alpha$	**M1**	Only allow cos/sin errors for T and W components, sign errors.
	Use $F = \mu R$ and combine to reach an equation in μ only.	**M1**	From part **(a)**, $F + T\cos\alpha = W\sin\alpha$ or $F = T\sin\alpha$.
	$\mu = \frac{1}{7}$	**A1**	
		3	

Question	Answer	Marks	Guidance
5(a)	For A: $F_A - T = m \times a\omega^2$	**M1**	Only allow sign errors.
	$F_A = \mu mg = \frac{1}{5}mg$	**B1**	Accept with g replaced by 10.
	Combine: $T = \frac{1}{5}mg - \frac{4}{25}mg$	**M1**	To reach an equation in T and mg only. Accept with g replaced by 10.
	$T = \frac{1}{25}mg$	**A1**	CAO
		4	
5(b)	For B: $F_B + T = km \times 2a\omega^2$	**M1**	Only allow sign errors.
	$F_B = \mu kmg = \frac{1}{5}kmg$ and combine to find k	**M1**	To reach an equation in k only.
	$k = \frac{1}{3}$	**A1**	
		3	

Question	Answer	Marks	Guidance
6(a)	$2\frac{\mathrm{d}v}{\mathrm{d}t} = -\frac{1}{10}(2v-1)^2 \mathrm{e}^{-t}$ so $\frac{\mathrm{d}v}{(2v-1)^2} = -\frac{1}{20}\mathrm{e}^{-t}\mathrm{d}t$ $\frac{p}{(2v-1)} = q\mathrm{e}^{-t} + A$	*M1	Separate variables and attempt to integrate both sides. Where p and q are constants.
	$-\frac{1}{2(2v-1)} = +\frac{1}{20}\mathrm{e}^{-t} + A$	A1	AEF
	$t = 0, v = 3,\quad A\left[= -\frac{3}{20}\right]$	DM1	Substituting the boundary condition and obtain a value.
	$v = \frac{1}{2} + \frac{5\mathrm{e}^t}{3\mathrm{e}^t - 1}$	*M1 A1	Find v in terms of t . AEF.
		5	
6(b)	Integrate: $x = pt + q\ln(r\mathrm{e}^{\pm t} - s)\ [+B]$	*M1	
	$x = \frac{1}{2}t + \frac{5}{3}\ln(3\mathrm{e}^t - 1)\ [+B]$	A1	AEF
	$t = 0, x = 1,\ B = 1 - \frac{5}{3}\ln 2$	DM1	Substituting the boundary condition and obtain a value.
	$x = 1 + \frac{1}{2}t + \frac{5}{3}\ln\frac{(3\mathrm{e}^t - 1)}{2}$	A1	AEF
		4	

Question	Answer	Marks	Guidance
7(a)	Energy: $\frac{1}{2}mu^2 = \frac{1}{2}mv^2 - mga(1-\cos\theta)$	*M1	m must be present, dimensionally correct, no missing terms. Allow sin instead of cos. Allow sign errors.
	N2L: $mg\cos\theta = \frac{mv^2}{a}$	B1	No reaction when P loses contact.
	Eliminate v^2	DM1	
	$\cos\theta = \frac{u^2+2ag}{3ag}$	A1	AG
		4	
7(b)	Vertical component of velocity of P when it leaves the sphere: $v\sin\theta$ $\left(=\sqrt{\frac{55ag}{216}}\right)$	*B1	Must not come from u.
	$V^2 = (v\sin\theta)^2 + 2g\times a(1+\cos\theta)$	DM1	Use of '$v^2 = u^2 + 2as$'. Allow $\sin\theta$ for $\cos\theta$. Allow sign errors.
	$V = \sqrt{\frac{847ag}{216}}$	A1	AEF
		3	

Question	Answer	Marks	Guidance
7(c)	$t = \frac{1}{g}\left(\sqrt{\frac{847ag}{216}} - \sqrt{\frac{55ag}{216}}\right)$	M1	
	$\frac{1}{6}\left(\sqrt{847} - \sqrt{55}\right)\sqrt{\frac{a}{6g}} = 1.48\sqrt{\frac{a}{g}}$	A1	
		2	

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Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

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FURTHER MATHEMATICS **9231/33**

Paper 3 Further Mechanics **May/June 2024**

1 hour 30 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.
- Where a numerical value for the acceleration due to gravity (g) is needed, use $10\,\mathrm{m\,s^{-2}}$.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Any blank pages are indicated.

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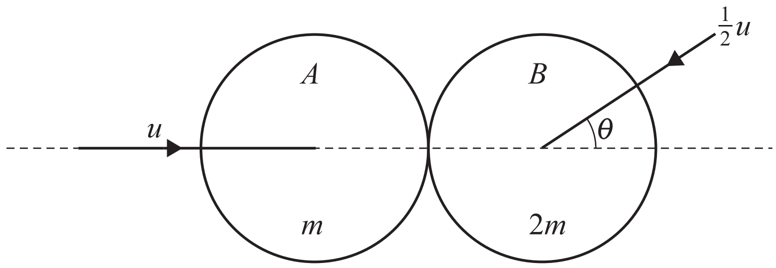

Two smooth uniform spheres A and B of equal radii have masses m and $2m$ respectively. The two spheres are moving on a smooth horizontal surface when they collide with speeds u and $\frac{1}{2}u$ respectively. Immediately before the collision, A's direction of motion is along the line of centres, and B's direction of motion makes an angle θ with the line of centres (see diagram).

As a result of the collision, the direction of motion of A is reversed and its speed is reduced to $\frac{1}{4}u$. The direction of motion of B again makes an angle θ with the line of centres, but on the opposite side of the line of centres. The speed of B is unchanged.

Find the value of the coefficient of restitution between the spheres. [4]

2 A particle P of mass m is attached to one end of a light elastic string of natural length a and modulus of elasticity $2mg$. A particle Q of mass km is attached to the other end of the string. Particle P lies on a smooth horizontal table. The string has part of its length in contact with the table and then passes through a small smooth hole H in the table.

Particle P moves in a horizontal circle on the surface of the table with constant speed $\sqrt{\frac{1}{2}ga}$. Particle Q hangs in equilibrium vertically below the hole with $HQ = \frac{1}{4}a$.

(a) Find, in terms of a, the extension in the string. [4]

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(b) Find the value of k. [2]

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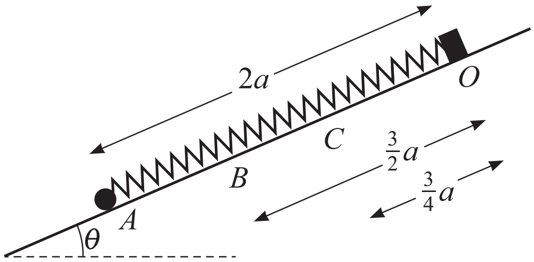
3 A particle P of mass m is attached to one end of a light inextensible string of length a. The other end of the string is attached to a fixed point O. When the particle is hanging vertically below O, it is projected horizontally with speed u so that it begins to move along a circular path. When P is at the lowest point of its motion, the tension in the string is T. When OP makes an angle θ with the upward vertical, the tension in the string is S.

(a) Show that $S = T - 3mg(1 + \cos\theta)$. [5]

(b) Given that $u = \sqrt{4ag}$, find the value of $\cos\theta$ when the string goes slack. [2]

4

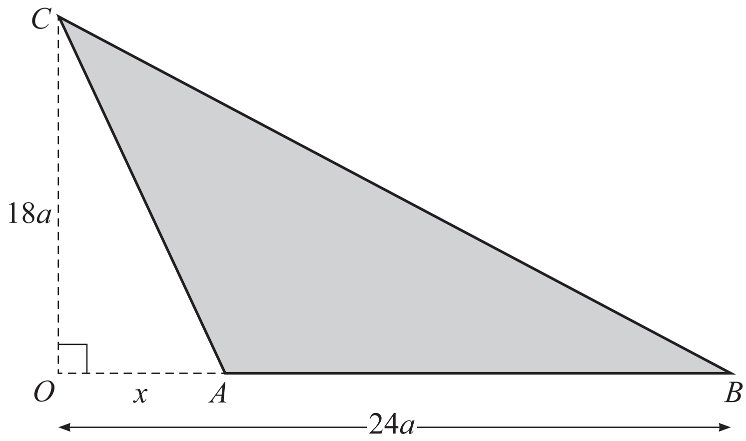


A light spring of natural length a and modulus of elasticity kmg is attached to a fixed point O on a smooth plane inclined to the horizontal at an angle θ, where $\sin\theta = \frac{3}{4}$. A particle of mass m is attached to the lower end of the spring and is held at the point A on the plane, where $OA = 2a$ and OA is along a line of greatest slope of the plane (see diagram).

The particle is released from rest and is moving with speed V when it passes through the point B on the plane, where $OB = \frac{3}{2}a$. The speed of the particle is $\frac{1}{2}V$ when it passes through the point C on the plane, where $OC = \frac{3}{4}a$.

Find the value of k. [7]

5



A uniform lamina is in the form of a triangle OBC, with $OC = 18a$, $OB = 24a$ and angle $COB = 90°$. The point A on OB is such that $OA = x$ (see diagram). The triangle OAC is removed from the lamina.

(a) Find, in terms of a and x, the distance of the centre of mass of the remaining object ABC from OC. [3]

The object ABC is suspended from C. In its equilibrium position, the side AB makes an angle θ with the vertical, where $\tan\theta = \frac{6}{5}$.

(b) Find x in terms of a. [4]

6 A particle P is projected with speed $u\ \mathrm{m\,s^{-1}}$ at an angle θ above the horizontal from a point O and moves freely under gravity. After 5 seconds the speed of P is $\frac{3}{4}u$.

(a) Show that $\frac{7}{16}u^2 - 100u\sin\theta + 2500 = 0$. [3]

(b) It is given that the velocity of P after 5 seconds is perpendicular to the initial velocity.

Find, in either order, the value of u and the value of $\sin\theta$. [5]

7 A parachutist of mass m kg opens his parachute when he is moving vertically downwards with a speed of $50\,\mathrm{m\,s^{-1}}$. At time t s after opening his parachute, he has fallen a distance x m from the point where he opened his parachute, and his speed is $v\,\mathrm{m\,s^{-1}}$. The forces acting on him are his weight and a resistive force of magnitude mv N.

(a) Find an expression for v in terms of t. [6]

(b) Find an expression for x in terms of t. [3]

(c) Find the distance that the parachutist has fallen, since opening his parachute, when his speed is $15\,\mathrm{m\,s^{-1}}$. [2]

Additional page

If you use the following page to complete the answer to any question, the question number must be clearly shown.

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Cambridge International AS & A Level

FURTHER MATHEMATICS **9231/33**

Paper 3 Further Mechanics **May/June 2024**

MARK SCHEME

Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2024 series for most Cambridge IGCSE, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptions for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1: Marks must be awarded in line with: • the specific content of the mark scheme or the generic level descriptors for the question • the specific skills defined in the mark scheme or in the generic level descriptors for the question • the standard of response required by a candidate as exemplified by the standardisation scripts.
GENERIC MARKING PRINCIPLE 2: Marks awarded are always **whole marks** (not half marks, or other fractions).
GENERIC MARKING PRINCIPLE 3: Marks must be awarded **positively**: • marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate • marks are awarded when candidates clearly demonstrate what they know and can do • marks are not deducted for errors • marks are not deducted for omissions • answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.
GENERIC MARKING PRINCIPLE 4: Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.
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WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	Along line of centres, PCLM: $-2m \times \frac{1}{2}u\cos\theta + mu = 2m \times \frac{1}{2}u\cos\theta - m \times \frac{1}{4}u$	**M1**	Masses must be included. Allow sign errors.
	$\cos\theta = \frac{5}{8}$	**A1**	
	NEL: $\frac{1}{2}u\cos\theta + \frac{1}{4}u = e\left(\frac{1}{2}u\cos\theta + u\right)$	**M1**	Allow sign errors, e must be on correct side.
	$e = \frac{3}{7}$	**A1**	AEF
		4	

Question	Answer	Marks	Guidance
2(a)	Hooke's law: $T = \frac{2mgx}{a}$	B1	$\frac{2mgx}{a}$ seen anywhere.
	$\rightarrow T = \dfrac{\frac{mga}{2}}{a - \frac{a}{4} + x}$	B1	RHS seen anywhere. May be in terms of radius or extended length, for example $\dfrac{\frac{mga}{2}}{l - \frac{1}{4}a}$, $\dfrac{\frac{mga}{2}}{r}$.
	Equate: $\frac{2mgx}{a} = \dfrac{\frac{mga}{2}}{\frac{3a}{4} + x}$, $4x^2 + 3ax - a^2 = 0$	M1	Equate two expressions for T and obtain a simplified homogeneous quadratic equation $4l^2 - 5al = 0$, $4r^2 - 3ar - a^2 = 0$, $2k^2 + 3k - 2 = 0$
	$x = \frac{a}{4}$	A1	Single correct answer only.
		4	
2(b)	$\uparrow T = kmg$	B1	kmg seen anywhere in an equation. This may be seen in part **(a)**. Note that no response in part **(b)** can earn B1 if kmg seen in part **(a)**.
	$T = \frac{2mgx}{a}$, $k = \frac{1}{2}$	B1	CWO. Part **(a)** needs to be correct.
		2	

Question	Answer	Marks	Guidance
3(a)	At lowest point, $T - mg = \frac{mu^2}{a}$	B1	Condone r used consistently instead of a throughout this question.
	When string makes angle θ with upward vertical, $S + mg\cos\theta = \frac{mv^2}{a}$	B1	
	Energy: $\frac{1}{2}mu^2 - \frac{1}{2}mv^2 = mga(1+\cos\theta)$	M1	Must include m. Allow $\sin\theta$ instead of $\cos\theta$ for this mark, allow sign errors.
	Eliminate u^2 and v^2	M1	Need to see at least one line of working.
	$S = T - 3mg(1+\cos\theta)$	A1	AG
		5	
3(b)	When string goes slack, $S = 0$ so $T = 3mg(1+\cos\theta)$	M1	May use $v^2 = ag\cos\theta$ substituted into energy equation.
	But $T = mg + \frac{mu^2}{a} = mg + 4mg = 5mg$, so $\cos\theta = \frac{2}{3}$	A1	
		2	

Question	Answer	Marks	Guidance
4	Consider one situation: A to B: Loss in EPE = $\frac{kmg}{2a}\times\left(a^2-\left(\frac{1}{2}a\right)^2\right) = \frac{3}{8}kmga$	B1	Accept unsimplified.
	Energy: $\frac{1}{2}mV^2+\frac{mga}{2}\sin\theta=\frac{3}{8}kmga\ \left(\left(V^2=\frac{3}{4}ga(k-1)\right)\right)$	M1A1	KE, GPE, EPE terms present. Must be dimensionally correct. Must include $\sin\theta$ or $\cos\theta$ in GPE.
	Consider a second situation: A to C: Loss in EPE = $\frac{kmg}{2a}\times\left(a^2-\left(-\frac{a}{4}\right)^2\right)=\frac{15}{32}kmga$	B1	Accept unsimplified.
	Energy: $\frac{1}{2}m\left(\frac{1}{2}V\right)^2+\frac{mg5a}{4}\sin\theta=\frac{15}{32}kmga\ \left(V^2=\frac{15}{4}ag(k-2)\right)$	M1	KE, GPE, EPE terms present. Must be dimensionally correct. Must include $\sin\theta$ or $\cos\theta$ in GPE.
	Third possible situation: B to C : Loss in EPE = $\frac{kmg}{2a}\times\left(\left(\frac{a}{2}\right)^2-\left(-\frac{a}{4}\right)^2\right)=\frac{3}{32}kmga$	(B1)	This may be used in combination with either of the first two situations. Mark to the candidate's benefit,
	Energy: $\frac{1}{2}m\left(\frac{1}{2}V\right)^2-\frac{1}{2}m\left(\frac{1}{4}V\right)^2-\frac{mg3a}{4}\sin\theta=-\frac{3}{32}kmga\ \left(V^2=\frac{1}{4}ag(6-k)\right)$	(M1)	KE, GPE, EPE terms present. Must be dimensionally correct . Must include $\sin\theta$ or $\cos\theta$ in GPE.
	Eliminate V^2 from two energy equations to obtain expression involving only k, a and possibly $\sin\theta$	M1	At least one of the energy equations must have scored M1.
	$k=\frac{9}{4}$	A1	
		7	

<table>
<tr><th>Question</th><th>Answer</th><th>Marks</th><th>Guidance</th></tr>
<tr><td>5(a)</td><td>
<table>
<tr><th></th><th>OBC</th><th>OAC</th><th>ABC</th></tr>
<tr><td>Area</td><td>$\frac{1}{2}\times 24a\times 18a$</td><td>$9ax$</td><td>$216a^2-9ax$</td></tr>
<tr><td>Centre of mass from OC</td><td>$8a$</td><td>$\frac{1}{3}x$</td><td>$\bar{x}$</td></tr>
</table>
Moments about OC
$\left(216a^2-9ax\right)\bar{x}=216a^2\times 8a-9ax\times\frac{1}{3}x$</td><td>M1 A1</td><td>Note that moments about OB is M0 ($\bar{y}=6a$).
Moments equation about OC with all terms present, allow sign error, dimensionally correct. All correct for A1.</td></tr>
<tr><td></td><td>$\bar{x}=\frac{576a^2-x^2}{72a-3x}$ or $\frac{x+24a}{3}$</td><td>A1</td><td>Accept any equivalent form.</td></tr>
<tr><td></td><td colspan="3">Alternative solution to question 5(a)</td></tr>
<tr><td></td><td>Consider system as equivalent to particles at (0, 18a), (x, 0) and (24a, 0)</td><td>B1</td><td></td></tr>
<tr><td></td><td>Then the x-coordinate of the centre of mass is at $\frac{1}{3}\left(x+24a\right)$</td><td>M1 A1</td><td></td></tr>
<tr><td></td><td></td><td>3</td><td></td></tr>
</table>

Question	Answer	Marks	Guidance
5(b)	$\tan\theta = \dfrac{18a-6a}{\bar{x}}$ or $\dfrac{18a-\bar{y}}{\bar{x}}$ or $\dfrac{\bar{y}}{15\text{a}-\bar{x}}$ or $\dfrac{18a}{12a+\frac{1}{2}x}$	M1 A1	Either way up (their value for $\bar{x}$ may be substituted in).
	$x^2-30ax+144a^2=0$ Or, with simplified form, $30a = x+24a$	M1	Obtain homogeneous (quadratic) equation Note that if simplified form of $\bar{x}$ is used, equation will be linear.
	$x=6a$	A1	Single correct answer only.
		4	

Question	Answer	Marks	Guidance
6(a)	Components of velocity are $u\cos\theta,\ u\sin\theta-gt$	B1	
	$(u\cos\theta)^2+(u\sin\theta-gt)^2=\left(\dfrac{3}{4}u\right)^2$	M1	Square and add components of velocity and equate to $\left(\dfrac{3}{4}u\right)^2$.
	$\dfrac{7}{16}u^2-100u\sin\theta+2500=0$	A1	AG At least one correct line of working seen.
		3	

Question	Answer	Marks	Guidance
6(b)	Let α be angle of direction of motion with horizontal at $t = 5$, then $(\tan\alpha =) \quad \frac{u\sin\theta - 5g}{u\cos\theta}$	B1	Either way up.
	$\tan\alpha\ \tan\theta = -1$, so $\tan\theta\left(\frac{u\sin\theta - 5g}{u\cos\theta}\right) = -1$	M1	Must be – 1 not +1. FT their expression for $\tan\alpha$.
	$u = 50\sin\theta$	A1	
	Use in result from part **(a)** to form equation in u or $\sin\theta$	M1	
	$u^2 = 1600,\ u = 40$ and $\sin\theta = \frac{4}{5}$	A1	Both.
	Alternative method for question 6(b)		
	$\rightarrow u\cos\theta = \frac{3}{4}u\sin\theta$	M1	
	$\tan\theta = \frac{4}{3}$ or $\sin\theta = \frac{4}{5}$	A1	
	$\uparrow \frac{3}{4}u\cos\theta = -\frac{4}{5}u + 50$	M1 A1	Allow sign error.
	$u = 40$ and $\sin\theta = \frac{4}{5}$	A1	Both seen.

Question	Answer	Marks	Guidance
6(b)	**Alternative method for question 6(b)**		
	$\rightarrow u\cos\theta = \frac{3}{4}u\sin\theta$	**M1**	
	$\tan\theta = \frac{4}{3}$ or $\sin\theta = \frac{4}{5}$	**A1**	
	Use in result from part **(a)** to form equation in u	**M1**	
	$u = 40 \left(\text{and } \frac{1000}{7}\right)$	**A1**	
	$u = 40(\text{only})$ and $\sin\theta = \frac{4}{5}$	**A1**	Both seen.
		5	

Question	Answer	Marks	Guidance
7(a)	$m\dfrac{dv}{dt} = m(10 - v)$	B1	No marks in this part if *suvat* used. Must have sight of m (for example in $F = ma$).
	$-\ln\lvert 10 - v\rvert = t + A$ or $-\ln(v - 10) = t + A$	*M1 A1	Separate variables and integrate to obtain a ln term. Constant may be omitted. Constant needed for A1
	Use $t = 0, v = 50$: $A = -\ln\lvert -40\rvert$	DM1	Find constant, dependent on previous M1. May use limits instead.
	$0.1ve^{t} = 4 + e^{t}$	M1	Remove all logs .
	$v = 10 + 40e^{-t}$	A1	Correct work only .
		6	
7(b)	$x = 10t - 40e^{-t} + B$	M1	No marks in this part if *suvat* used in part **(a)** or part **(b)**. Integrate their answer to part **(a)**. Constant may be omitted.
	Use $t = 0, x = 0$: $B = 40$	M1	Use initial condition in their expression for x in terms of t .
	$x = 10t - 40e^{-t} + 40$	A1	
		3	

Question	Answer	Marks	Guidance
7(c)	When $v = 15, e^{-t} = \frac{1}{8}$, $t = 2.08$ or ln 8	**M1**	No marks in this part if *suvat* used in part **(a)** part **(b)** or part **(c)**. Find value of t from their answer to part **(a)**.
	$x = 55.8$ (metres)	**A1**	Note 35 + 10 ln 8 scores A0.
		2	

Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER | CANDIDATE NUMBER

8795483048

FURTHER MATHEMATICS **9231/31**

Paper 3 Further Mechanics **October/November 2023**

1 hour 30 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS
- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.
- Where a numerical value for the acceleration due to gravity (g) is needed, use $10\,\mathrm{m\,s^{-2}}$.

INFORMATION
- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Any blank pages are indicated.

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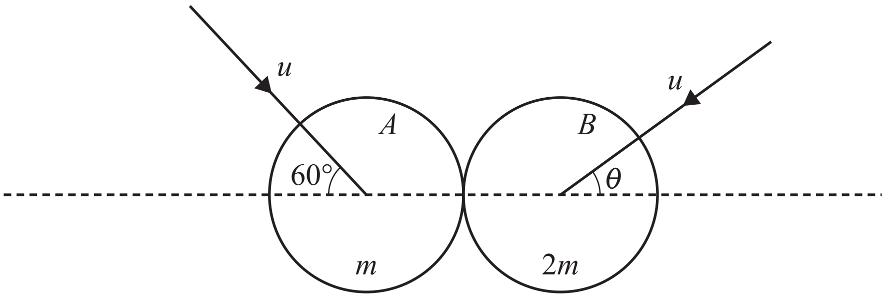
1



Two uniform smooth spheres A and B of equal radii have masses m and $2m$ respectively. The two spheres are moving with equal speeds u on a smooth horizontal surface when they collide. Immediately before the collision, A's direction of motion makes an angle of 60° with the line of centres, and B's direction of motion makes an angle θ with the line of centres (see diagram). The coefficient of restitution between the spheres is e.

After the collision, the component of the velocity of A along the line of centres is v and B moves perpendicular to the line of centres. Sphere A now has twice as much kinetic energy as sphere B.

(a) Show that $v = \frac{1}{2}u(4\cos\theta - 1)$. [1]

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(b) Find the value of $\cos\theta$. [4]

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(c) Find the value of e. [2]

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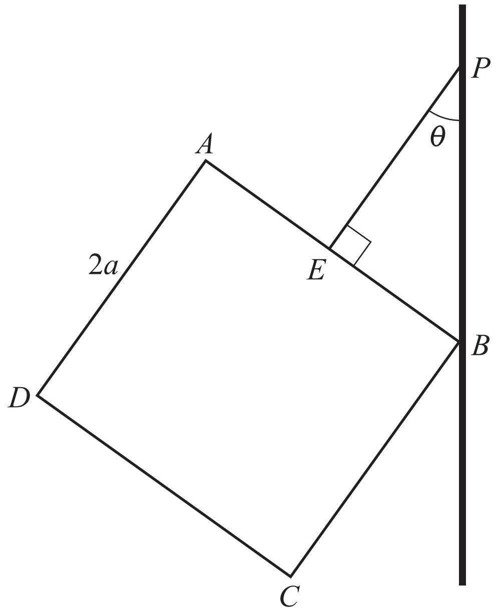
2 A ball of mass 2 kg is projected vertically downwards with speed $5\,\mathrm{m\,s^{-1}}$ through a liquid. At time t s after projection, the velocity of the ball is $v\,\mathrm{m\,s^{-1}}$ and its displacement from its starting point is x m. The forces acting on the ball are its weight and a resistive force of magnitude $0.2v^2$ N.

(a) Find an expression for v in terms of t. [6]

(b) Deduce what happens to v for large values of t. [1]

3



A uniform square lamina of side $2a$ and weight W is suspended from a light inextensible string attached to the midpoint E of the side AB. The other end of the string is attached to a fixed point P on a rough vertical wall. The vertex B of the lamina is in contact with the wall. The string EP is perpendicular to the side AB and makes an angle θ with the wall (see diagram). The string and the lamina are in a vertical plane perpendicular to the wall. The coefficient of friction between the wall and the lamina is $\frac{1}{2}$.

Given that the vertex B is about to slip up the wall, find the value of $\tan\theta$. [8]

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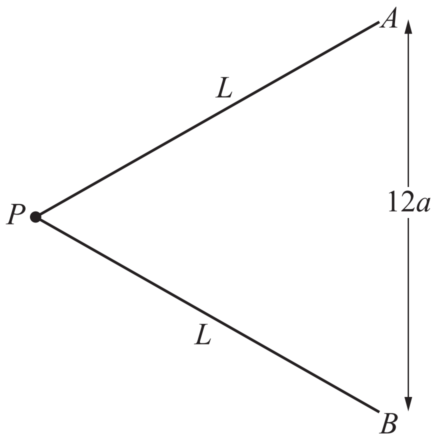

A light elastic string has natural length $8a$ and modulus of elasticity $5mg$. A particle P of mass m is attached to the midpoint of the string. The ends of the string are attached to points A and B which are a distance $12a$ apart on a smooth horizontal table. The particle P is held on the table so that $AP = BP = L$ (see diagram). The particle P is released from rest. When P is at the midpoint of AB it has speed $\sqrt{80ag}$.

(a) Find L in terms of a. [5]

(b) Find the initial acceleration of P in terms of g. [3]

5 A particle P is projected with speed $u \text{ m s}^{-1}$ at an angle θ above the horizontal from a point O on a horizontal plane and moves freely under gravity. During its flight P passes through the point which is a horizontal distance $3a$ from O and a vertical distance $\frac{3}{8}a$ above the horizontal plane. It is given that $\tan\theta = \frac{1}{3}$.

(a) Show that $u^2 = 8ag$. [2]

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A particle Q is projected with speed $V \text{ m s}^{-1}$ at an angle α above the horizontal from O at the instant when P is at its highest point. Particles P and Q both land at the same point on the horizontal plane at the same time.

(b) Find V in terms of a and g. [7]

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6 A particle P of mass m is attached to one end of a light inextensible rod of length $3a$. An identical particle Q is attached to the other end of the rod. The rod is smoothly pivoted at a point O on the rod, where $OQ = x$. The system, of rod and particles, rotates about O in a vertical plane.

At an instant when the rod is vertical, with P above Q, the particle P is moving horizontally with speed u. When the rod has turned through an angle of 60° from the vertical, the speed of P is $2\sqrt{ag}$, and the tensions in the two parts of the rod, OP and OQ, have equal magnitudes.

(a) Show that the speed of Q when the rod has turned through an angle of 60° from the vertical is $\dfrac{2x}{3a-x}\sqrt{ag}$. [2]

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(b) Find x in terms of a. [5]

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(c) Find u in terms of a and g. [4]

Additional page

If you use the following page to complete the answer to any question, the question number must be clearly shown.

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Cambridge Assessment International Education

Cambridge International AS & A Level

FURTHER MATHEMATICS **9231/11**

Paper 1 Further Pure Mathematics 1 **October/November 2023**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2023 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

This document consists of **15** printed pages.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:
Marks must be awarded in line with:
- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:
Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:
Marks must be awarded **positively**:
- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:
Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics-Specific Marking Principles

1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.

2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.

3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.

4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).

5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.

6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1(a)	$r^2+2r+1-r^2=2r+1$	**B1**	Expands
	$2\sum_{r=1}^{n} r+n=(n+1)^2-1^2$	**M1 A1**	Uses method of differences and sums both sides.
	$\Rightarrow 2\sum_{r=1}^{n} r=n^2+n=n(n+1)$	**A1**	AG.
		4	
1(b)	$\sum_{r=1}^{n}(r+a)=\sum_{r=1}^{n} r+an$	**M1**	Relates with $\sum r$.
	$\frac{1}{2}n(n+1)+an=n$	**M1**	Applies $\sum_{r=1}^{n} r=\frac{1}{2}n(n+1)$.
	$a=\frac{1}{2}(1-n)$	**A1**	
		3	

Question	Answer	Marks	Guidance
2	$1=\frac{1-2x+x^2}{(1-x)^2}=\frac{(1-x)^2}{(1-x)^2}$ so H_1 is true.	B1	Checks base case.
	Assume that $\sum_{r=1}^{k} rx^{r-1}=\frac{1-(k+1)x^k+kx^{k+1}}{(1-x)^2}$.	B1	States inductive hypothesis.
	$\sum_{r=1}^{k+1} rx^{r-1}=\frac{1-(k+1)x^k+kx^{k+1}}{(1-x)^2}+(k+1)x^k$	M1	Considers sum to $k+1$.
	$\frac{1-(k+1)x^k+kx^{k+1}+(k+1)x^k\left(1-2x+x^2\right)}{(1-x)^2}$	M1	Puts over a common denominator.
	$\frac{1+kx^{k+1}+(k+1)x^k\left(-2x+x^2\right)}{(1-x)^2}=\frac{1-(k+2)x^{k+1}+(k+1)x^{k+2}}{(1-x)^2}$	A1	
	So H_{k+1} is true. By induction, H_n is true for all positive integers *n*.	A1	States conclusion.
		6	

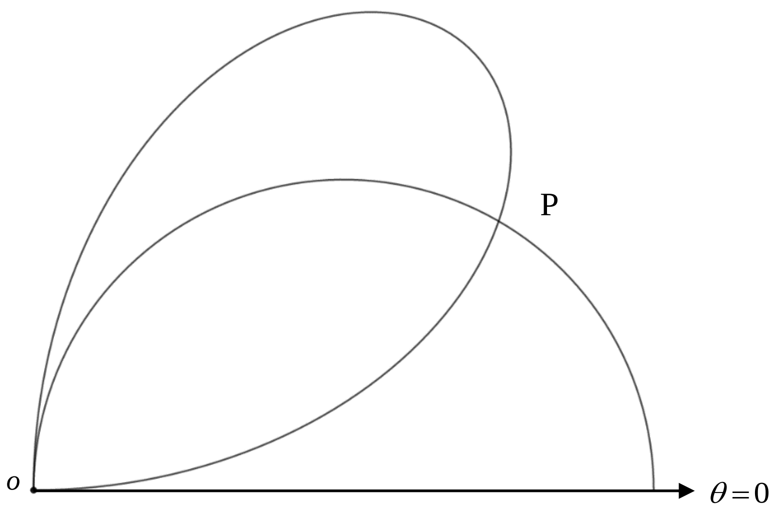
Question	Answer	Marks	Guidance
3(a)	$b = -(\alpha + \beta + \gamma + \delta) = -3$	**B1**	
	$5 = (-3)^2 - 2(\alpha\beta + \alpha\gamma + \alpha\delta + \beta\gamma + \beta\delta + \gamma\delta)$	**M1 A1**	Uses formula for sum of squares.
	$c = 2$	**A1**	
	$6 = \dfrac{\alpha\beta\gamma + \beta\gamma\delta + \gamma\delta\alpha + \delta\alpha\beta}{\alpha\beta\gamma\delta} = \dfrac{-d}{-2}$	**M1**	Uses $\alpha^{-1} + \beta^{-1} + \gamma^{-1} + \delta^{-1} = \dfrac{\alpha\beta\gamma + \beta\gamma\delta + \gamma\delta\alpha + \delta\alpha\beta}{\alpha\beta\gamma\delta}$.
	$d = 12$	**A1**	Equation is $x^4 - 3x^3 + 2x^2 + 12x - 2 = 0$.
		6	
3(b)	$\alpha^4 + \beta^4 + \gamma^4 + \delta^4 = 3(-27) - 2(5) - 12(3) + 2(4)$	**M1**	Uses *their* quartic equation derived in **(a)**.
	–119	**A1**	
		2	

Question	Answer	Marks	Guidance
4(a)	$\begin{pmatrix}2\\-2\\3\end{pmatrix}-\begin{pmatrix}-2\\-3\\-5\end{pmatrix}=\begin{pmatrix}4\\1\\8\end{pmatrix}$	**B1**	Finds direction of one line to another.
	$\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\-4 & 3 & 5\\2 & -3 & 1\end{vmatrix}=\begin{pmatrix}18\\14\\6\end{pmatrix}\sim\begin{pmatrix}9\\7\\3\end{pmatrix}$	**M1 A1**	Find common perpendicular.
	$\frac{1}{\sqrt{139}}\left\|\begin{pmatrix}4\\1\\8\end{pmatrix}\bullet\begin{pmatrix}9\\7\\3\end{pmatrix}\right\|=\frac{67}{\sqrt{139}}(=5.68)$	**M1 A1**	Uses formula for shortest distance.
		5	
4(b)	$\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\1 & 0 & 1\\-4 & 3 & 5\end{vmatrix}=\begin{pmatrix}3\\9\\-3\end{pmatrix}\sim\begin{pmatrix}1\\3\\-1\end{pmatrix}$	**M1 A1**	Finds vector perpendicular to the plane.
	$1(-1)+3(-3)-1(-4)=-6\Rightarrow x+3y-z=-6$	**M1 A1**	Uses point in the plane.
		4	

Question	Answer	Marks	Guidance
5(a)	$\begin{vmatrix}1 & 3\\2 & 5\end{vmatrix} - k\begin{vmatrix}2 & 3\\3 & 5\end{vmatrix} + 3\begin{vmatrix}2 & 1\\3 & 2\end{vmatrix} = 0 \Rightarrow -1-k+3=0 \Rightarrow k=2$	**M1 A1**	Sets determinant of **A** equal to zero.
	$\begin{pmatrix}-2 & -1 & 1\\1 & 1 & 3\end{pmatrix}\begin{pmatrix}1 & 2 & 3\\2 & 1 & 3\\3 & 2 & 5\end{pmatrix}\begin{pmatrix}0 & -2\\-1 & 3\\0 & 0\end{pmatrix} = \begin{pmatrix}-2 & -1 & 1\\1 & 1 & 3\end{pmatrix}\begin{pmatrix}-2 & 4\\-1 & -1\\-2 & 0\end{pmatrix}$	**M1**	Multiplying two matrices correctly, correct dimensions.
	$\begin{pmatrix}3 & -7\\-9 & 3\end{pmatrix}$	**M1 A1**	Completing matrix multiplication, AG.
		5	
5(b)	$\begin{pmatrix}3 & -7\\-9 & 3\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}3x-7y\\-9x+3y\end{pmatrix}$	**B1**	Transforms $\begin{pmatrix}x\\y\end{pmatrix}$ to $\begin{pmatrix}X\\Y\end{pmatrix}$.
	$-9x+3mx = m(3x-7mx)$	**M1 A1**	Uses $y=mx$ and $Y=mX$.
	$-9+3m = 3m-7m^2 \Rightarrow 7m^2 = 9$	**A1**	
	$y=\frac{3}{\sqrt{7}}x$ and $y=-\frac{3}{\sqrt{7}}x$	**A1**	
		5	

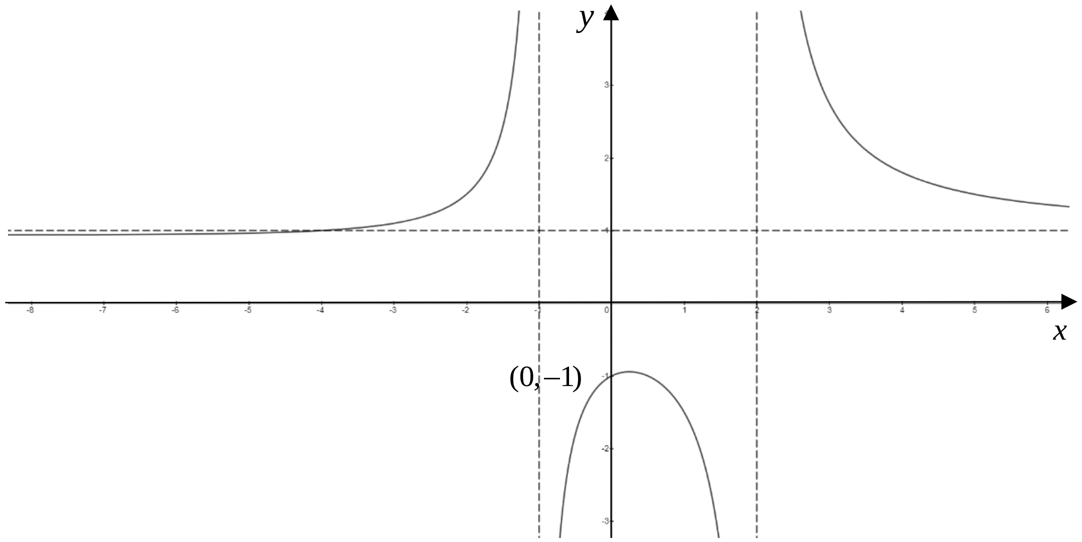
Question	Answer	Marks	Guidance
5(c)	$\mathbf{D}=\begin{pmatrix}\alpha & 0\\ 0 & \alpha\end{pmatrix}$	B1	
	$\mathbf{E}=\begin{pmatrix}\beta & 0\\ 0 & 1\end{pmatrix}$	B1	
	$\mathbf{F}=\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}$	B1	
	$\begin{pmatrix}3 & -7\\ -9 & 3\end{pmatrix}=\begin{pmatrix}\alpha & 0\\ 0 & \alpha\end{pmatrix}-9\begin{pmatrix}0 & \beta\\ 1 & 0\end{pmatrix}$	M1	Setting up simultaneous equations using their **D** and **E**.
	$\mathbf{D}=\begin{pmatrix}3 & 0\\ 0 & 3\end{pmatrix}$ $\mathbf{E}=\begin{pmatrix}\frac{7}{9} & 0\\ 0 & 1\end{pmatrix}$	A1	Condone $\alpha=3, \beta=\frac{7}{9}$ if it is clear that they refer to the correct matrices.
		5	

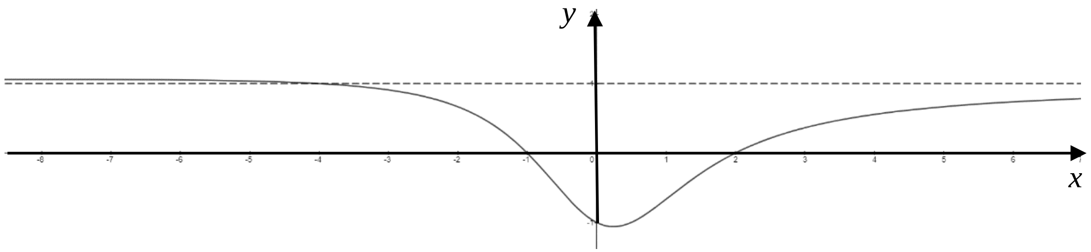
Question	Answer	Marks	Guidance
6(a)	$x^2-x+\frac{1}{4}+y^2=\frac{1}{4}\Rightarrow r^2-r\cos\theta+\frac{1}{4}=\frac{1}{4}$	B1	Uses $x^2+y^2=r^2$ and $x=r\cos\theta$.
	$r\left(r-\cos\theta\right)=0$	M1	Factorises.
	$\left[r\neq 0\Rightarrow\right]r=\cos\theta$	A1	AG.
		3	

Question	Answer	Marks	Guidance
6(b)	$\sin 2\theta = \cos\theta \Rightarrow 2\sin\theta\cos\theta = \cos\theta$	M1	Sets r values equal and uses $\sin 2\theta = 2\sin\theta\cos\theta$.
	$\cos\theta \neq 0 \Rightarrow \sin\theta = \frac{1}{2}$	A1	$\cos\theta \neq 0$ must be recognised.
	$\left(\frac{1}{2}\sqrt{3}, \frac{1}{6}\pi\right)$	A1	
		3	
6(c)		B1	Initial line drawn and one curve correct.
		B1	Other curve correct.
		B1	Intersection marked in correct position and both curves labelled.
		3	

Question	Answer	Marks	Guidance
6(d)	$\frac{1}{2}\int_0^{\frac{1}{6}\pi}\sin^2 2\theta\,d\theta+\frac{1}{2}\int_{\frac{1}{6}\pi}^{\frac{1}{2}\pi}\cos^2\theta\,d\theta$	M1	Uses $\frac{1}{2}\int r^2\,d\theta$ with correct limits.
	$\frac{1}{2}\int_0^{\frac{1}{6}\pi}\sin^2 2\theta\,d\theta=\frac{1}{4}\int_0^{\frac{1}{6}\pi}1-\cos 4\theta\,d\theta$	M1	Integrates $\sin^2 2\theta$ using identity.
	$=\frac{1}{4}\left[\theta-\frac{1}{4}\sin 4\theta\right]_0^{\frac{1}{6}\pi}$	A1	
	$\frac{1}{2}\int_{\frac{1}{6}\pi}^{\frac{1}{2}\pi}\cos^2\theta\,d\theta=\frac{1}{4}\int_{\frac{1}{6}\pi}^{\frac{1}{2}\pi}1+\cos 2\theta\,d\theta$	M1	Integrates $\cos^2\theta$ using identity.
	$=\frac{1}{4}\left[\theta+\frac{1}{2}\sin 2\theta\right]_{\frac{1}{6}\pi}^{\frac{1}{2}\pi}$	A1	
	$\frac{1}{4}\left(\frac{1}{6}\pi-\frac{1}{8}\sqrt{3}\right)+\frac{1}{4}\left(\frac{1}{2}\pi-\frac{1}{6}\pi-\frac{1}{4}\sqrt{3}\right)=\frac{1}{8}\left(\pi-\frac{3}{4}\sqrt{3}\right)$	A1	
		6	

Question	Answer	Marks	Guidance
7(a)	$x=-1,\quad x=2$	B1	Vertical asymptotes.
	$y=1$	B1	Horizontal asymptote.
		2	

Question	Answer	Marks	Guidance
7(b)	$\frac{dy}{dx} = \frac{(x^2-x-2)(2x)-(x^2+2)(2x-1)}{\left(x^2-x-2\right)^2}$	**M1***	Finds $\frac{dy}{dx}$.
	$x^2+8x-2=0$	**DM1**	Sets equal to 0 and forms equation.
	$(-8.2, 0.9)$, $(0.2, -0.9)$.	**A1 A1**	Condone $\left(-4-3\sqrt{2}, \frac{2}{3}\sqrt{2}\right)$, $\left(-4+3\sqrt{2}, -\frac{2}{3}\sqrt{2}\right)$.
		4	
7(c)		**B1**	Axes and all three asymptotes.
		B1	Correct shape and position, crossing horizontal asymptote.
		B1	States $(0,-1)$ coordinates of intersection with axes, may be seen on diagram.
		3	

Question	Answer	Marks	Guidance
7(d)		B1 FT	FT from sketch in **(c)**
		B1	All correct.
		2	
7(e)	$\frac{x^2+2}{x^2-x-2}=1$ or $\frac{x^2+2}{x^2-x-2}=-1$ $x+4=0$ or $2x^2-x=0$	M2	Finds critical points, award M1 for each case.
	$x=-4$ or $x=0$, $x=\frac{1}{2}$	A1	
	$-4<x<-1$, $0<x<\frac{1}{2}$, $x>2$	B1	Must have three distinct regions. Condone $\leqslant -1$ and $\geqslant 2$.
		4	

Cambridge Assessment
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CANDIDATE NAME				
CENTRE NUMBER			CANDIDATE NUMBER	

1394054656

FURTHER MATHEMATICS **9231/32**

Paper 3 Further Mechanics **October/November 2023**

1 hour 30 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.
- Where a numerical value for the acceleration due to gravity (g) is needed, use $10\,\mathrm{m\,s^{-2}}$.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Any blank pages are indicated.

DC (RW/CGW) 316270/2

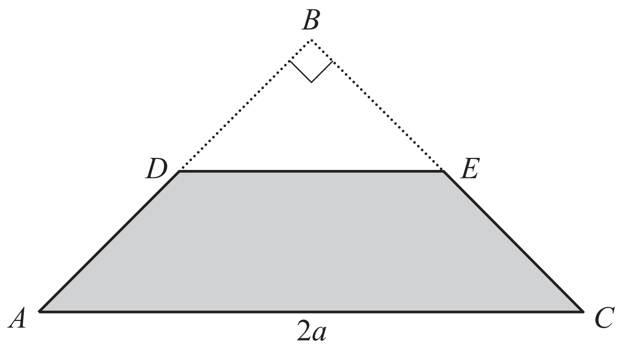
1 One end of a light inextensible string of length a is attached to a fixed point O. The other end of the string is attached to a particle of mass m. The string is taut and makes an angle θ with the downward vertical through O, where $\cos\theta = \frac{2}{3}$. The particle moves in a horizontal circle with speed v.

Find v in terms of a and g. [4]

2 A particle P of mass 0.5 kg moves in a straight line. At time t s the velocity of P is $v\,\mathrm{m\,s^{-1}}$ and its displacement from a fixed point O on the line is x m. The only forces acting on P are a force of magnitude $\frac{150}{(x+1)^2}$ N in the direction of increasing displacement and a resistive force of magnitude $\frac{450}{(x+1)^3}$ N. When $t = 0$, $x = 0$ and $v = 20$.

Find v in terms of x, giving your answer in the form $v = \frac{Ax+B}{(x+1)}$, where A and B are constants to be determined. [6]

3



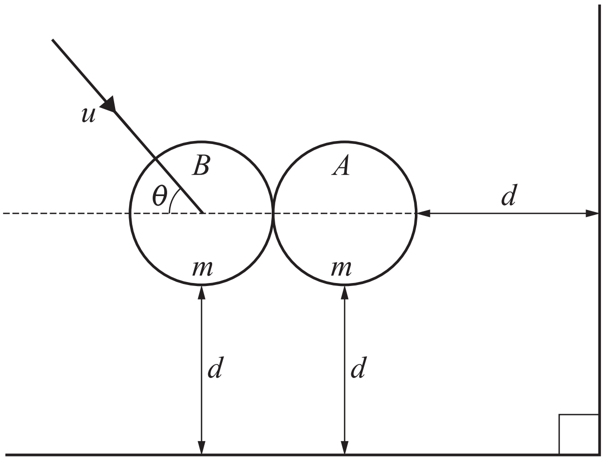
A uniform lamina is in the form of an isosceles triangle ABC in which $AC = 2a$ and angle $ABC = 90°$. The point D on AB is such that the ratio $DB : AB = 1 : k$. The point E on CB is such that DE is parallel to AC. The triangle DBE is removed from the lamina (see diagram).

(a) Find, in terms of k, the distance of the centre of mass of the remaining lamina $ADEC$ from the midpoint of AC. [4]

When the lamina *ADEC* is freely suspended from the vertex *A*, the edge *AC* makes an angle θ with the downward vertical, where $\tan\theta = \frac{5}{18}$.

(b) Find the value of *k*. [3]

4



Two smooth vertical walls meet at right angles. The smooth sphere A, with mass m, is at rest on a smooth horizontal surface and is at a distance d from each wall. An identical smooth sphere B is moving on the horizontal surface with speed u at an angle θ with the line of centres when the spheres collide (see diagram). After the collision, the spheres take the same time to reach a wall. The coefficient of restitution between the spheres is $\frac{1}{2}$.

(a) Find the value of $\tan\theta$. [4]

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(b) Find the percentage loss in the total kinetic energy of the spheres as a result of this collision. [3]

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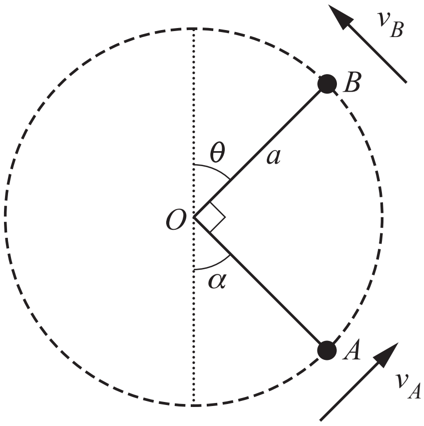

A bead of mass m moves on a smooth circular wire, with centre O and radius a, in a vertical plane. The bead has speed v_A when it is at the point A where OA makes an angle α with the downward vertical through O, and $\cos\alpha = \frac{3}{5}$. Subsequently the bead has speed v_B at the point B, where OB makes an angle θ with the upward vertical through O. Angle AOB is a right angle (see diagram). The reaction of the wire on the bead at B is in the direction OB and has magnitude equal to $\frac{1}{6}$ of the magnitude of the reaction when the bead is at A.

(a) Find, in terms of m and g, the magnitude of the reaction at B. [6]

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(b) Given that $v_A = \sqrt{kag}$, find the value of k. [2]

6 A particle P is projected with speed u at an angle α above the horizontal from a point O on a horizontal plane and moves freely under gravity. The horizontal and vertical displacements of P from O at a subsequent time t are denoted by x and y respectively.

(a) Derive the equation of the trajectory of P in the form

$$y = x\tan\alpha - \frac{gx^2}{2u^2}\sec^2\alpha.$$ [3]

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During its flight, P must clear an obstacle of height h m that is at a horizontal distance of 32 m from the point of projection. When $u = 40\sqrt{2}\,\mathrm{m\,s^{-1}}$, P just clears the obstacle. When $u = 40\,\mathrm{m\,s^{-1}}$, P only achieves 80% of the height required to clear the obstacle.

(b) Find the two possible values of h. [6]

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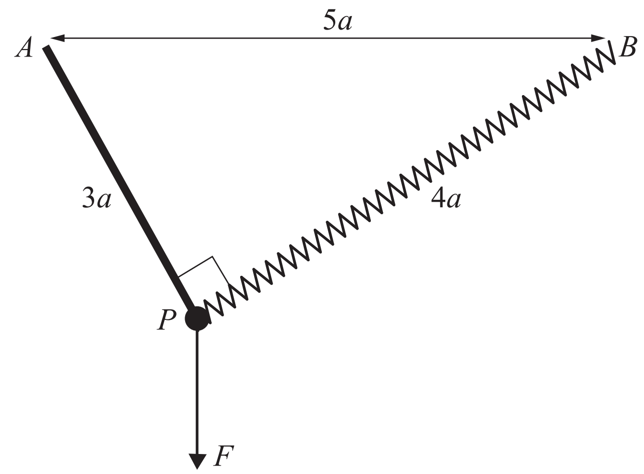

A particle P of mass m is attached to one end of a light rod of length $3a$. The other end of the rod is able to pivot smoothly about the fixed point A. The particle is also attached to one end of a light spring of natural length a and modulus of elasticity kmg. The other end of the spring is attached to a fixed point B. The points A and B are in a horizontal line, a distance $5a$ apart, and these two points and the rod are in a vertical plane.

Initially, P is held in equilibrium by a vertical force F with the stretched length of the spring equal to $4a$ (see diagram). The particle is released from rest in this position and has a speed of $\frac{6}{5}\sqrt{2ag}$ when the rod becomes horizontal.

(a) Find the value of k. [5]

(b) Find F in terms of m and g. [2]

(c) Find, in terms of m and g, the tension in the rod immediately before it is released. [2]

Additional page

If you use the following page to complete the answer to any question, the question number must be clearly shown.

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Cambridge Assessment International Education

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FURTHER MATHEMATICS **9231/12**

Paper 1 Further Pure Mathematics 1 **October/November 2023**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2023 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

This document consists of **15** printed pages.

[Turn over

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1: Marks must be awarded in line with: • the specific content of the mark scheme or the generic level descriptors for the question • the specific skills defined in the mark scheme or in the generic level descriptors for the question • the standard of response required by a candidate as exemplified by the standardisation scripts.
GENERIC MARKING PRINCIPLE 2: Marks awarded are always **whole marks** (not half marks, or other fractions).
GENERIC MARKING PRINCIPLE 3: Marks must be awarded **positively**: • marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate • marks are awarded when candidates clearly demonstrate what they know and can do • marks are not deducted for errors • marks are not deducted for omissions • answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.
GENERIC MARKING PRINCIPLE 4: Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.
GENERIC MARKING PRINCIPLE 5: Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).
GENERIC MARKING PRINCIPLE 6: Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics-Specific Marking Principles

1. Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2. Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3. Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5. Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.
6. Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate’s intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1(a)	$\frac{1}{2}n(n+1)(2n+1)+\frac{3}{2}n(n+1)+n$	M1 A1	Substitutes correct formulae from MF19.
	n^3+3n^2+3n	A1	Simplifies
		3	
1(b)	$\frac{1}{r^3}-\frac{1}{(r+1)^3}=\frac{(r+1)^3-r^3}{r^3(r+1)^3}=\frac{r^3+3r^2+3r+1-r^3}{r^3(r+1)^3}=\frac{3r^2+3r+1}{r^3(r+1)^3}$	M1 A1	Puts over a common denominator and expands, AG.
	$\sum_{r=1}^{n}\frac{3r^2+3r+1}{r^3(r+1)^3}=\sum_{r=1}^{n}\left(\frac{1}{r^3}-\frac{1}{(r+1)^3}\right)$ $=1-\frac{1}{2^3}+\frac{1}{2^3}-\frac{1}{3^3}+\ldots+\frac{1}{n^3}-\frac{1}{(n+1)^3}$	M1 A1	Shows three complete terms, including last.
	$1-\frac{1}{(n+1)^3}$	A1	
		5	
1(c)	1	B1FT	FT from *their* answer to part **(b)**.
		1	

Question	Answer	Marks	Guidance
2	$\frac{\mathrm{d}}{\mathrm{d}x}\left(x^2\mathrm{e}^x\right)=x^2\mathrm{e}^x+2x\mathrm{e}^x=\left(x^2+2x\right)\mathrm{e}^x$ so true when $n=1$.	**M1 A1**	Differentiates once using the product rule.
	Assume that $\frac{\mathrm{d}^k}{\mathrm{d}x^k}\left(x^2\mathrm{e}^x\right)=\left(x^2+2kx+k(k-1)\right)\mathrm{e}^x$ [for some value of k].	**B1**	States inductive hypothesis.
	$\frac{\mathrm{d}^{k+1}}{\mathrm{d}x^{k+1}}\left(x^2\mathrm{e}^x\right)=\left(x^2+2kx+k(k-1)\right)\mathrm{e}^x+\mathrm{e}^x\left(2x+2k\right)$	**M1**	Differentiates kth derivative.
	$\left(x^2+2(k+1)x+k(k+1)\right)\mathrm{e}^x$	**A1**	
	So true when $n=k+1$. By induction, true for all positive integers n.	**A1**	States conclusion.
		6	

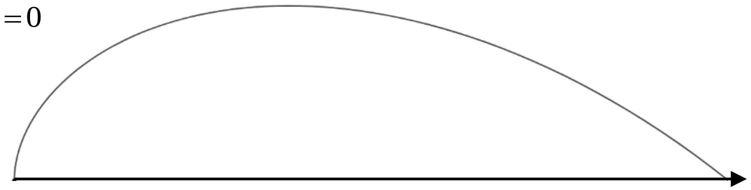
Question	Answer	Marks	Guidance
3(a)	Shear followed by a stretch.	**B2**	Award B1 if given in the wrong order.
		2	
3(b)	$\lvert OPQR\rvert=\lvert\det\mathbf{M}\rvert=\lvert k\rvert$	**B1**	
	$\mathbf{M}^{-1}=\frac{1}{k}\begin{pmatrix}1 & 0\\ -1 & k\end{pmatrix}$	**M1 A1**	
		3	

Question	Answer	Marks	Guidance
3(c)	$\begin{pmatrix} k & 0 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} x \\ \frac{1}{k-1}x \end{pmatrix}$	**B1**	Sets $y = \frac{1}{k-1}x$.
	$\begin{pmatrix} k & 0 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} x \\ \frac{1}{k-1}x \end{pmatrix} = \begin{pmatrix} kx \\ x+\frac{1}{k-1}x \end{pmatrix} = \begin{pmatrix} kx \\ \frac{k}{k-1}x \end{pmatrix}$	**M1**	Shows that $Y = \frac{1}{k-1}X$.
	$k\begin{pmatrix} x \\ \frac{1}{k-1}x \end{pmatrix}$	**A1**	
	Alternative method for 3(c)		
	$\begin{pmatrix} k & 0 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} kx \\ x+y \end{pmatrix}$	**B1**	Transforms $\begin{pmatrix} x \\ y \end{pmatrix}$ to $\begin{pmatrix} X \\ Y \end{pmatrix}$
	$X = kx$ and $mX = x+y$ $mkx = x + mx$	**M1**	Uses $y = mx$ and $Y = mX$
	$m = \dfrac{1}{k-1}$ $y = \dfrac{1}{k-1}x$	**A1**	AG
		3	

Question	Answer	Marks	Guidance
4(a)	$y = 3x+1 \Rightarrow x = \frac{1}{3}(y-1)$ $\Rightarrow 27\left(\frac{y-1}{3}\right)^3 + 18\left(\frac{y-1}{3}\right)^2 + 6\left(\frac{y-1}{3}\right) - 1 = 0$	**B1**	Substitutes.
	$\Rightarrow (y-1)^3 + 2(y-1)^2 + 2(y-1) - 1 = 0$ $\Rightarrow y^3 - 3y^2 + 3y - 1 + 2y^2 - 4y + 2 + 2y - 2 - 1 = 0$	**M1**	Expands.
	$y^3 - y^2 + y - 2 = 0$	**A1**	AG.
		3	
4(b)	$S_2 = 1^2 - 2(1) = -1$	**M1 A1**	Uses formula for sum of squares, AG.
	$S_3 = (3\alpha+1)^3 + (3\beta+1)^3 + (3\gamma+1)^3 = -1 - (1) + 6$	**M1**	Uses $y^3 = y^2 - y + 2$ or expands and uses original equation.
	4	**A1**	
		4	
4(c)	$S_{-1} = \frac{(3\alpha+1)(3\beta+1) + (3\beta+1)(3\gamma+1) + (3\gamma+1)(3\alpha+1)}{(3\alpha+1)(3\beta+1)(3\gamma+1)} = \frac{1}{2}$	**B1**	
	$2S_{-2} = S_1 - 3 + S_{-1} = 1 - 3 + \frac{1}{2}$	**M1**	Uses $2y^{-2} = y - 1 + y^{-1}$.
	$S_{-2} = -\frac{3}{4}$	**A1**	CAO
		3	

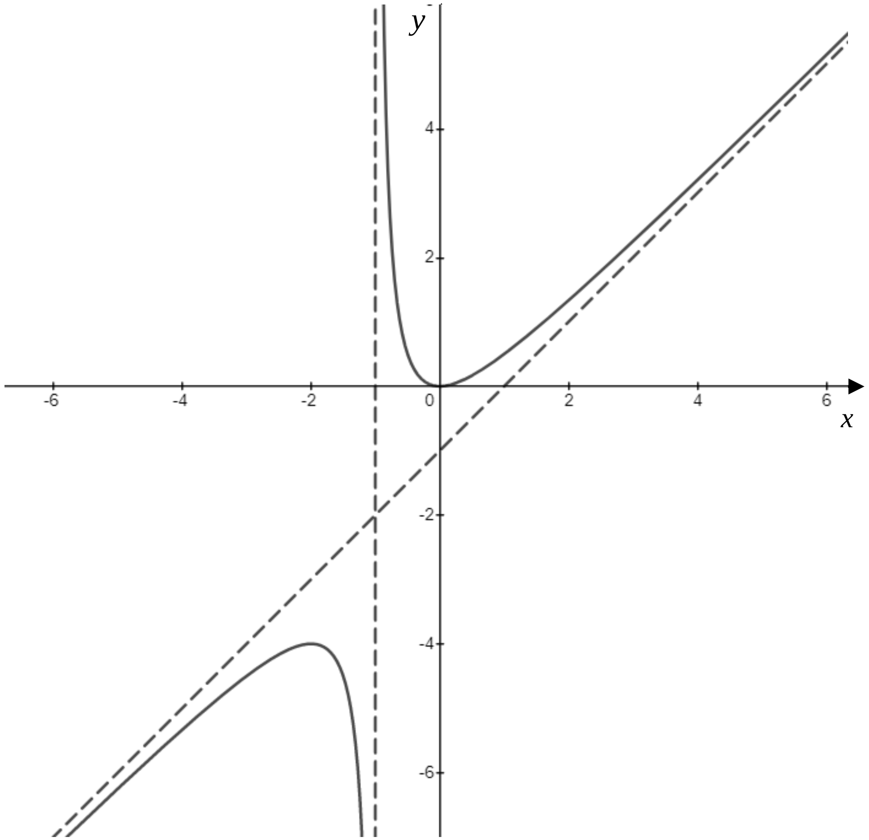
Question	Answer	Marks	Guidance
5(a)	$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -2 & -3 \\ 3 & 0 & -1 \end{vmatrix} = \begin{pmatrix} 2 \\ -8 \\ 6 \end{pmatrix} \sim \begin{pmatrix} 1 \\ -4 \\ 3 \end{pmatrix}$	**M1 A1**	Finds perpendicular to Π_1.
	$1(1)-4(-1)+3(-2)=-1$	**M1**	Uses point on Π_1.
	$x-4y+3z=-1$	**A1**	
		4	
5(b)	$\begin{pmatrix} 1 \\ -4 \\ 3 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = 1-4+3=0$	**M1 A1**	Shows dot product with direction of line is 0.
		2	
5(c)	$\frac{1}{\sqrt{1^2+4^2+3^2}}\begin{pmatrix} -4 \\ 1 \\ 3 \end{pmatrix}\bullet\begin{pmatrix} 1 \\ -4 \\ 3 \end{pmatrix}$ or $\frac{1}{\sqrt{1^2+4^2+3^2}}\left(\begin{pmatrix} -3 \\ 0 \\ 1 \end{pmatrix}\bullet\begin{pmatrix} 1 \\ -4 \\ 3 \end{pmatrix}+1\right)$	**M1 A1**	Uses correct formula for distance from point on l to Π_1. $\frac{1}{\sqrt{1^2+4^2+3^2}}\left(-3.1+0.-4+1.3+1\right)$
	$\frac{1}{\sqrt{26}}(=0.196)$	**A1**	
		3	

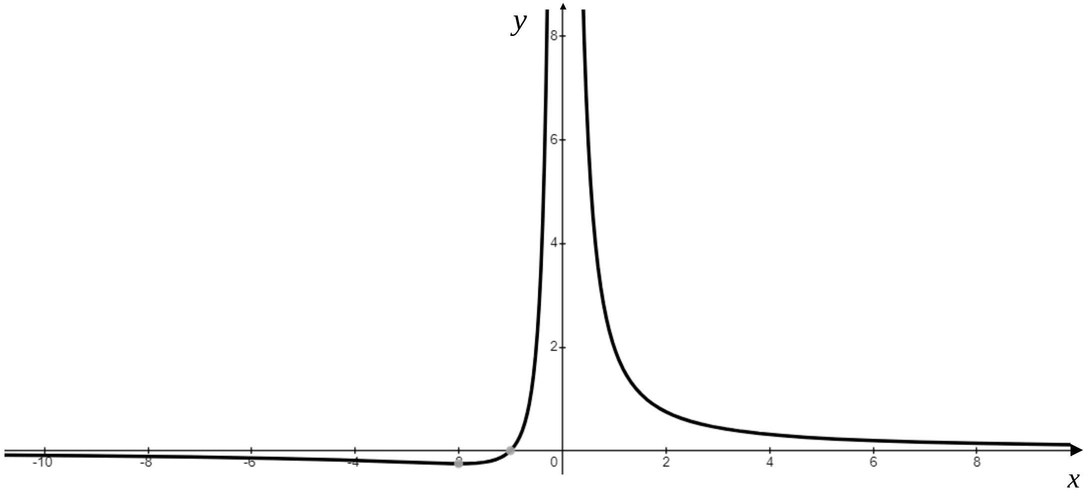
Question	Answer	Marks	Guidance
5(d)	States point common to both planes e.g. $\begin{pmatrix} \frac{1}{15} \\ \frac{4}{15} \\ 0 \end{pmatrix}$.	**B1**	$\begin{pmatrix} \frac{5}{7} \\ 0 \\ \frac{-4}{7} \end{pmatrix}$ or $\begin{pmatrix} 0 \\ \frac{5}{17} \\ \frac{1}{17} \end{pmatrix}$ or alternative.
	$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -4 & 3 \\ 3 & 3 & 2 \end{vmatrix} = \begin{pmatrix} -17 \\ 7 \\ 15 \end{pmatrix}$	**M1 A1**	Finds direction of line.
	$\mathbf{r} = \begin{pmatrix} \frac{5}{7} \\ 0 \\ -\frac{4}{7} \end{pmatrix} + \lambda \begin{pmatrix} -17 \\ 7 \\ 15 \end{pmatrix}$	**A1**	OE.
		4	

Question	Answer	Marks	Guidance
6(a)	$\theta=0$ 	B1	Initial line drawn. Correct shape, r strictly decreasing.
		B1	Correct shape at extremities.
	$1-\mathrm{e}^{-\frac{1}{2}\pi}$	B1	May be seen on *their* diagram.
		3	
6(b)	$\frac{1}{2}\int_0^{\frac{1}{2}\pi}\left(\mathrm{e}^{-\theta}-\mathrm{e}^{-\frac{1}{2}\pi}\right)^2\mathrm{d}\theta$	M1	Uses correct formula with correct limits.
	$\frac{1}{2}\int_0^{\frac{1}{2}\pi}\mathrm{e}^{-2\theta}-2\mathrm{e}^{-\theta-\frac{1}{2}\pi}+\mathrm{e}^{-\pi}\,\mathrm{d}\theta$	A1	
	$\frac{1}{2}\left[-\frac{1}{2}\mathrm{e}^{-2\theta}+2\mathrm{e}^{-\theta-\frac{1}{2}\pi}+\mathrm{e}^{-\pi}\theta\right]_0^{\frac{1}{2}\pi}$	M1 A1	Integrates.
	$\frac{1}{2}\left(-\frac{1}{2}\mathrm{e}^{-\pi}+2\mathrm{e}^{-\pi}+\frac{1}{2}\pi\mathrm{e}^{-\pi}+\frac{1}{2}-2\mathrm{e}^{-\frac{1}{2}\pi}\right)=\frac{3}{4}\mathrm{e}^{-\pi}+\frac{1}{4}\pi\mathrm{e}^{-\pi}-\mathrm{e}^{-\frac{1}{2}\pi}+\frac{1}{4}$	A1	
		5	

Question	Answer	Marks	Guidance
6(c)	$y=\left(\mathrm{e}^{-\theta}-\mathrm{e}^{-\frac{1}{2}\pi}\right)\sin\theta$	B1	Uses $y=r\sin\theta$
	$\frac{\mathrm{d}y}{\mathrm{d}\theta}=\left(\mathrm{e}^{-\theta}-\mathrm{e}^{-\frac{1}{2}\pi}\right)\cos\theta+\sin\theta\left(-\mathrm{e}^{-\theta}\right)=0$	M1 A1	Sets derivative equal to zero.
	$\left[\theta\neq\frac{1}{2}\pi\Rightarrow\right]1+\left(\frac{-\mathrm{e}^{-\theta}}{\mathrm{e}^{-\theta}-\mathrm{e}^{-\frac{1}{2}\pi}}\right)\tan\theta=0\Rightarrow1-\mathrm{e}^{\theta-\frac{1}{2}\pi}-\tan\theta=0$	A1	AG.
	$1-\mathrm{e}^{0.56-\frac{1}{2}\pi}-\tan0.56=0.00912$ and $1-\mathrm{e}^{0.57-\frac{1}{2}\pi}-\tan0.57=-0.00856$	B1	Shows sign change.
		5	

Question	Answer	Marks	Guidance
7(a)	$x=-1$	B1	Vertical asymptote.
	$y=\frac{(x+1)(x-1)+1}{x+1}$	M1	Oblique asymptote.
	$y=x-1$	A1	
		3	
7(b)	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{x^2+2x}{(x+1)^2}=0$	M1	Sets $\frac{\mathrm{d}y}{\mathrm{d}x}=0$.
	$(0,0)$, $(-2,-4)$	A1	
		2	

Question	Answer	Marks	Guidance
7(c)		B1	Axes and asymptotes.
		B1	Left branch correct.
		B1	Right branch correct.
		3	
7(d)	$\left(-2,-\frac{1}{4}\right)$	B1 B1	B1 for each correct coordinate. SC B1 for $\left(-2,-\frac{1}{4}\right)$ and $(0,0)$.
		2	

Question	Answer	Marks	Guidance
7(e)		B1	Left branch correct.
		B1	Right branch correct.
	$\frac{x^2}{x+1}=1$ or $\frac{x^2}{x+1}=-1$ $x^2-x-1=0$	M2	Finds critical points, award M1 for each case.
	$x=\frac{1}{2}-\frac{1}{2}\sqrt{5}$ or $x=\frac{1}{2}+\frac{1}{2}\sqrt{5}$	A1	
	$x<-1,\ \frac{1}{2}-\frac{1}{2}\sqrt{5}<x<\frac{1}{2}+\frac{1}{2}\sqrt{5},\ \ x\neq 0$	B1	Condone missing $x\neq 0$.
		6	

Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

6081966353

FURTHER MATHEMATICS **9231/32**

Paper 3 Further Mechanics **May/June 2023**

1 hour 30 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.
- Where a numerical value for the acceleration due to gravity (g) is needed, use $10\,\text{m}\,\text{s}^{-2}$.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Any blank pages are indicated.

DC (CE/SG) 316268/2

1 One end of a light elastic string, of natural length a and modulus of elasticity $3mg$, is attached to a fixed point O. The other end of the string is attached to a particle P of mass m. The string hangs with P vertically below O. The particle P is pulled vertically downwards so that the extension of the string is $2a$. The particle P is then released from rest.

(a) Find the speed of P when it is at a distance $\frac{3}{4}a$ below O. [3]

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(b) Find the initial acceleration of P when it is released from rest. [2]

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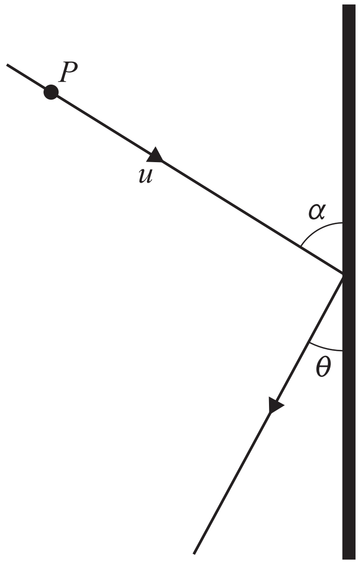

A particle P of mass m is moving with speed u on a fixed smooth horizontal surface. It collides at an angle α with a fixed smooth vertical barrier. After the collision, P moves at an angle θ with the barrier, where $\tan\theta = \frac{1}{2}$ (see diagram). The coefficient of restitution between P and the barrier is e. The particle P loses 20% of its kinetic energy as a result of the collision.

Find the value of e. [5]

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3 A particle P of mass m is attached to one end of a light inextensible string of length a. The other end of the string is attached to a fixed point O. The particle P is held at the point A, where OA makes an angle θ with the downward vertical through O, and with the string taut. The particle P is projected perpendicular to OA in an upwards direction with speed u. It then starts to move along a circular path in a vertical plane. The string goes slack when P is at B, where angle AOB is 90° and the speed of P is $\sqrt{\frac{4}{5}ag}$.

(a) Find the value of $\sin\theta$. [2]

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(b) Find, in terms of m and g, the tension in the string when P is at A. [5]

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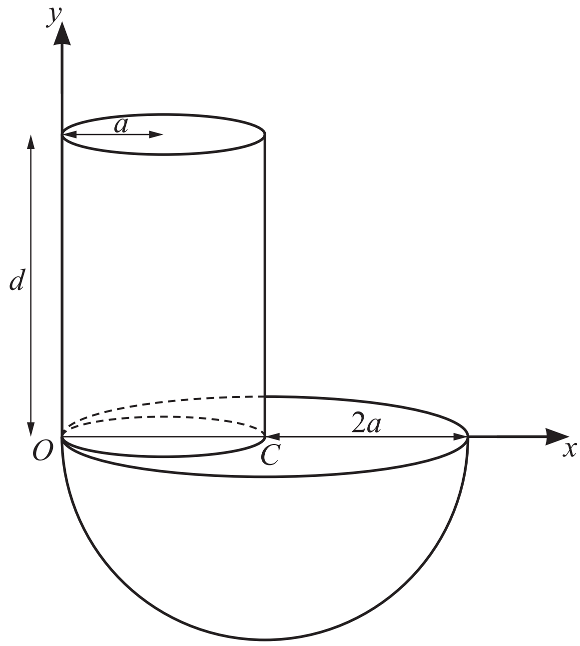

An object is formed from a solid hemisphere, of radius $2a$, and a solid cylinder, of radius a and height d. The hemisphere and the cylinder are made of the same material. The cylinder is attached to the plane face of the hemisphere. The line OC forms a diameter of the base of the cylinder, where C is the centre of the plane face of the hemisphere and O is common to both circumferences (see diagram). Relative to axes through O, parallel and perpendicular to OC as shown, the centre of mass of the object is $(\bar{x}, \bar{y})$.

(a) Show that $\bar{x} = \dfrac{32a^2 + 3ad}{16a + 3d}$ and find an expression, in terms of a and d, for $\bar{y}$. [5]

The object is placed on a rough plane which is inclined to the horizontal at an angle θ where $\sin\theta = \frac{1}{6}$. The object is in equilibrium with CO horizontal, where CO lies in a vertical plane through a line of greatest slope.

(b) Find d in terms of a. [3]

5 A light elastic string of natural length a and modulus of elasticity λmg has one end attached to a fixed point O on a smooth horizontal surface. When a particle of mass m is attached to the free end of the string, it moves with speed v in a horizontal circle with centre O and radius x. When, instead, a particle of mass $2m$ is attached to the free end of the string, this particle moves with speed $\frac{1}{2}v$ in a horizontal circle with centre O and radius $\frac{3}{4}x$.

(a) Find x in terms of a. [5]

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(b) Given that $v = \sqrt{12ag}$, find the value of λ. [2]

6 A particle P moving in a straight line has displacement x m from a fixed point O on the line and velocity $v\,\mathrm{m\,s^{-1}}$ at time t s. The acceleration of P, in $\mathrm{m\,s^{-2}}$, is given by $6v\sqrt{v+9}$. When $t = 0$, $x = 2$ and $v = 72$.

(a) Find an expression for v in terms of x. [4]

(b) Find an expression for x in terms of t. [5]

7 At time t s, a particle P is projected with speed $40\,\mathrm{m\,s^{-1}}$ at an angle θ above the horizontal from a point O on a horizontal plane and moves freely under gravity. The greatest height achieved by P during its flight is H m and the corresponding time is T s.

(a) Obtain expressions for H and T in terms of θ. [2]

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During the time between $t = T$ and $t = 3$, P descends a distance $\frac{1}{4}H$.

(b) Find the value of θ. [4]

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(c) Find the speed of P when $t = 3$. [3]

Additional page

If you use the following page to complete the answer to any question, the question number must be clearly shown.

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Cambridge Assessment International Education

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FURTHER MATHEMATICS **9231/32**

Paper 3 Further Mechanics 32 **May/June 2023**

MARK SCHEME

Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2023 series for most Cambridge IGCSE, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:
Marks must be awarded in line with:
- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:
Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:
Marks must be awarded **positively**:
- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:
Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1(a)	$\frac{3mg}{2a}(2a)^2$	**B1**	Correct EPE term seen
	$\frac{1}{2}mv^2 + mg \times \left(3a - \frac{3}{4}a\right) = \frac{3mg}{2a}(2a)^2$	**M1**	Dimensionally correct energy equation. Must have one KE, one EPE term and at least one GPE. Allow sign errors.
	$v = \sqrt{\frac{15}{2}ag} \quad \left[2.74\sqrt{ag}\right]$	**A1**	AEF
		3	
1(b)	$T - mg = mA$ and $T = \frac{3mg}{a} \times 2a$	**M1**	N2L and Hooke's law
	Acceleration $= 5g$ [upwards]	**A1**	Allow ± 50 or $\pm 5g$
		2	

Question	Answer	Marks	Guidance
2	Parallel to wall $v\cos\theta = u\cos\alpha$ Perpendicular to wall $v\sin\theta = eu\sin\alpha$	M1	Both
	Dividing, $e = \dfrac{1}{2\tan\alpha}$	A1	AEF
	KE reduced by 20%, so $\frac{1}{2}mu^2\left(\cos^2\alpha + e^2\sin^2\alpha\right) = \frac{4}{5}\times\frac{1}{2}mu^2$	M1	Dimensionally correct equation in u or v, but not both. Must have either α or θ, but not both. Must see $\frac{4}{5}$ on the correct side of the equation.
	Eliminate e: $\cos\alpha = \dfrac{4}{5}$	A1	
	$e = \dfrac{2}{3}$	A1	

Question	Answer	Marks	Guidance
2	**Alternative method for question 2**		
	Parallel to wall $v\cos\theta = u\cos\alpha$ Perpendicular to wall $v\sin\theta = eu\sin\alpha$	**M1**	Both
	$\left[\sin(\theta)=\frac{\sqrt{5}}{5},\ \cos(\theta)=\frac{2\sqrt{5}}{5}\right]$ $u\sin(\alpha)=\frac{\sqrt{5}v}{5e},\ u\cos(\alpha)=\frac{2\sqrt{5}v}{5}$	**A1**	
	$u^2=\left[u^2cos^2(\alpha)+u^2sin^2(\alpha)\right]=\frac{4v^2}{5}+\frac{v^2}{5e^2}$	**A1**	AEF, e.g. $\frac{v^2}{5}\left(4+\frac{1}{e^2}\right)$
	$\frac{1}{2}mv^2=\left[\frac{4}{5}\times\frac{1}{2}mu^2=\right]\frac{2}{5}m\frac{v^2}{5}\left(4+\frac{1}{e^2}\right)$	**M1**	Dimensionally correct equation in v. Must have either α or θ, but not both. Must see $\frac{4}{5}$ or $\frac{2}{5}$ on the correct side of the equation.
	$e=\frac{2}{3}$	**A1**	
		5	

Question	Answer	Marks	Guidance
3(a)	At B, $mg\sin\theta = \frac{m4ag}{5a}$	**M1**	Allow cos instead of sin for M1. Do not award until tension $=0$ used. Mass must be seen. No sign error.
	$\sin\theta = \frac{4}{5}$	**A1**	
		2	
3(b)	At A, $T - mg\cos\theta = \frac{mu^2}{a}$	**B1**	
	Energy $\frac{1}{2}mu^2 - \frac{1}{2}m\times\frac{4ag}{5} = mga(\cos\theta + \sin\theta)$	**M1 A1**	Energy equation with 4 terms, dimensionally correct. Mass must be present, allow sign errors. Must see $\frac{1}{2}$ in the KE terms.
	Solve to find T	**M1**	Complete method leading to an expression in mg for T.
	$T = \frac{21}{5}mg$	**A1**	CWO
		5	

Question	Answer	Marks	Guidance
4(a)	[Mass is proportional to volume] <table><tr><th></th><th>Volume</th><th>Distance of centre of mass from vertical axis</th><th>Distance of centre of mass from *OC*</th></tr><tr><td>Hemisphere</td><td>$\frac{2}{3}\pi(2a)^3$</td><td>$2a$</td><td>$-\frac{3}{8}\times 2a$</td></tr><tr><td>Cylinder</td><td>$\pi a^2 d$</td><td>a</td><td>$\frac{1}{2}d$</td></tr><tr><td>Object</td><td>$\frac{2}{3}\pi(2a)^3+\pi a^2 d$</td><td>$\bar{x}$</td><td>$\bar{y}$</td></tr></table> $\left(\frac{2}{3}\pi(2a)^3+\pi a^2 d\right)\bar{x}=\frac{16}{3}\pi a^3\times 2a+\pi a^2 d\times a$	**M1 A1**	Moments equation, dimensionally correct, correct number of terms. Allow sign errors.
	Simplify to $\bar{x}=\frac{32a^2+3ad}{16a+3d}$	**A1**	AG. At least one line of intermediate working.
	$\left(\frac{2}{3}\pi(2a)^3+\pi a^2 d\right)\bar{y}=\frac{16}{3}\pi a^3\times\left(-\frac{3}{8}\times 2a\right)+\pi a^2 d\times\frac{1}{2}d$	**M1**	Moments equation, dimensionally correct, correct number of terms. Allow sign errors.
	$\bar{y}=\frac{3(d^2-8a^2)}{2(16a+3d)}$	**A1**	AEF
		5	

Question	Answer	Marks	Guidance
4(b)	$\sin\theta = \dfrac{2a - \bar{x}}{2a}$	**B1**	
	$2a \times \dfrac{1}{6} = 2a - \dfrac{32a^2 + 3ad}{16a + 3d}$ $\dfrac{5}{3}(16a + 3d) = (32a + 3d)$	**M1**	Remove fractions
	$d = \dfrac{8}{3}a$	**A1**	
		3	

Question	Answer	Marks	Guidance
5(a)	Hooke's law, $T_1 = \frac{\lambda mg}{a}(x-a)$ or $T_2 = \frac{\lambda mg}{a}\left(\frac{3}{4}x-a\right)$	B1	
	Also, $T_1 = \frac{mv^2}{x}$ and equate $\frac{mv^2}{x} = \frac{\lambda mg}{a}(x-a)$	M1	$v^2 = \frac{\lambda gx(x-a)}{a}$ Dimensionally correct terms.
	Similarly: $\frac{\lambda mg\left(\frac{3x}{4}-a\right)}{a} = \frac{2m\left(\frac{1}{2}v\right)^2}{\frac{3}{4}x}$	M1	$v^2 = \frac{3\lambda gx}{2a}\left(\frac{3}{4}x-a\right)$ Must have $\frac{1}{2}v$ and $\frac{3x}{4}$ on the RHS. Their dimensionally correct T_2.
	Equate expressions for v^2 and solve for x in terms of a.	M1	
	$x = 4a$	A1	WWW
		5	**SC B3** for answer of $4a$ using λ instead of λmg.
5(b)	$\lambda = \frac{a}{xg(x-a)}v^2$ or $\lambda = \frac{2a}{3xg\left(\frac{3}{4}x-a\right)}v^2$ and substitute $x = 4a$, $v = \sqrt{12ag}$	M1	FT their expression for x.
	1	A1	CAO
		2	

Question	Answer	Marks	Guidance
6(a)	$v\dfrac{dv}{dx}=6v\sqrt{v+9}$ and attempt to separate variables and integrate	M1	
	$2\sqrt{v+9}=6x+A$	A1	
	$x=2, v=72 \quad A=6$	M1	Use initial condition to find constant.
	$v=9(x+1)^2-9$	A1	Correct, AEF.
		4	
6(b)	$\left[\dfrac{dx}{dt}=9\left(x^2+2x\right),\ \dfrac{dx}{x(x+2)}=9dt\right]\ \dfrac{1}{2}\left(\dfrac{1}{x}-\dfrac{1}{x+2}\right)dx=9dt$	M1	Separate variables and write in the form $\left(\dfrac{a}{x}-\dfrac{b}{x+c}\right)dx=dt$
	$\dfrac{1}{2}\ln\left(\dfrac{x}{x+2}\right)=9t+B$	A1	Integrate, any correct form.
	$t=0, x=2 \quad B=\dfrac{1}{2}\ln\dfrac{1}{2}$	M1	Use initial condition to find constant.
	$18t=\ln\left(\dfrac{2x}{x+2}\right) \quad e^{18t}=\dfrac{2x}{x+2}$	M1	Take logarithms
	$x=\dfrac{2e^{18t}}{2-e^{18t}}$ or $x=\dfrac{2}{2e^{-18t}-1}$	A1	Any correct form
		5	

Question	Answer	Marks	Guidance
7(a)	$H = 80\sin^2\theta$ or $\dfrac{800\sin^2\theta}{g}$	B1	
	$T = 4\sin\theta$ or $\dfrac{40\sin\theta}{g}$	B1	
		2	
7(b)	Between $t = T$ and $t = 3$ $\uparrow$ $\frac{1}{4}H = \frac{1}{2}\times 10\times(3-T)^2$	M1 A1	No extra terms.
	Use results from part **(a)** $\frac{1}{4}80\sin^2\theta = 5(3-4\sin\theta)^2$ $4\sin^2\theta - 8\sin\theta + 3 = 0$	M1	Substitute their expressions for H and T from part **(a)** and obtain a quadratic equation in $\sin\theta$ with no more than three terms.
	$\sin\theta = \frac{1}{2}$, $\theta = 30°$	A1	Single answer. NFWW.
	Alternative method for question 7 part (b)		
	$\frac{3}{4}H = y(3) = 40\times 3\ \sin\theta - \frac{1}{2}\times 10\times 3^2$	M1 A1	$120\ \sin\theta - 45$
	Use results from (a): $\frac{3}{4}80\sin^2\theta = 120\ \sin\theta - 45$ $4\sin^2\theta - 8\sin\theta + 3 = 0$	M1	Substitute their expressions for H and T from part **(a)** and obtain a quadratic equation in $\sin\theta$ with no more than three terms.
	$\sin\theta = \frac{1}{2}$, $\theta = 30°$	A1	Single answer. NFWW.
		4	

Question	Answer	Marks	Guidance
7(c)	When $t = 3$ speeds $\rightarrow 40\cos\theta$ and $\uparrow 40\sin\theta - 10\times 3$	**B1**	
	Square and add to find square of speed: $v^2 = \left(20\sqrt{3}\right)^2 + (-10)^2$	**M1**	Must be numerical.
	$v^2 = 1300, \quad v = 10\sqrt{13}$ [= 36.1]	**A1**	
		3	

Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER | CANDIDATE NUMBER

1370453044

FURTHER MATHEMATICS **9231/33**

Paper 3 Further Mechanics **May/June 2023**

1 hour 30 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.
- Where a numerical value for the acceleration due to gravity (g) is needed, use $10\,\text{m}\,\text{s}^{-2}$.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Any blank pages are indicated.

DC (CE/SG) 316276/1

1 A particle P of mass m is attached to one end of a light inextensible string of length a. The other end of the string is attached to a fixed point O. The particle P is held at the point A, where OA makes an angle α with the downward vertical through O, and with the string taut. The particle P is projected perpendicular to OA in an upwards direction with speed $\sqrt{3ag}$. It then starts to move along a circular path in a vertical plane. The string goes slack when P is at B, where OB makes an angle θ with the upward vertical.

Given that $\cos\alpha = \frac{4}{5}$, find the value of $\cos\theta$. [4]

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2 One end of a light elastic string, of natural length a and modulus of elasticity λmg, is attached to a fixed point O. The string lies on a smooth horizontal surface. A particle P of mass m is attached to the other end of the string. The particle P is projected in the direction OP. When the length of the string is $\frac{4}{3}a$, the speed of P is $\sqrt{2ag}$. When the length of the string is $\frac{5}{3}a$, the speed of P is $\frac{1}{2}\sqrt{2ag}$.

Find the value of λ. [4]

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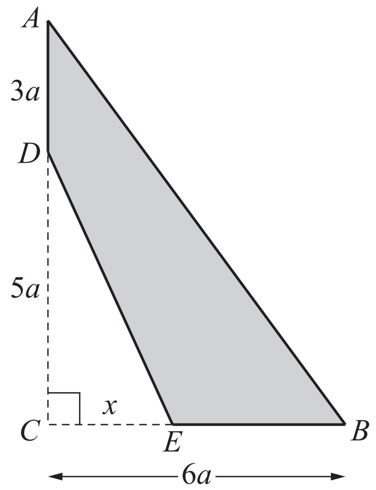


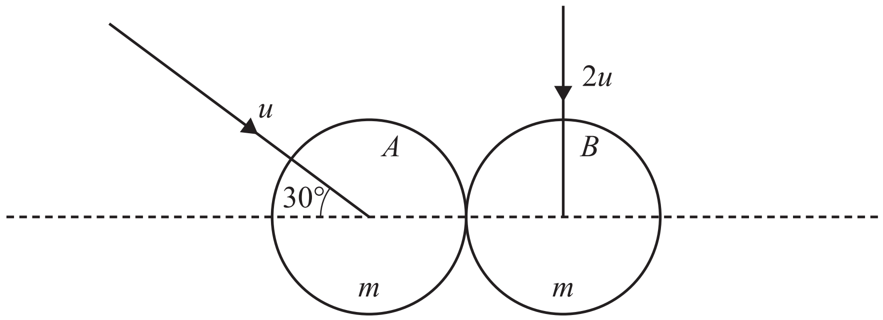
A uniform lamina is in the form of a triangle ABC, with $AC = 8a$, $BC = 6a$ and angle $ACB = 90°$. The point D on AC is such that $AD = 3a$. The point E on CB is such that $CE = x$ (see diagram). The triangle CDE is removed from the lamina.

(a) Find, in terms of a and x, the distance of the centre of mass of the remaining object $ADEB$ from AC. [4]

The object $ADEB$ is on the point of toppling about the point E when the object is in the vertical plane with its edge EB on a smooth horizontal surface.

(b) Find x in terms of a. [3]

4



Two identical smooth uniform spheres A and B each have mass m. The two spheres are moving on a smooth horizontal surface when they collide with speeds u and $2u$ respectively. Immediately before the collision, A's direction of motion makes an angle of 30° with the line of centres, and B's direction of motion is perpendicular to the line of centres (see diagram). After the collision, A and B are moving in the same direction. The coefficient of restitution between the spheres is e.

(a) Find the value of e. [5]

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(b) Find the loss in the total kinetic energy of the spheres as a result of the collision. [3]

5 One end of a light elastic string, of natural length $12a$ and modulus of elasticity kmg, is attached to a fixed point O. The other end of the string is attached to a particle of mass m. The particle moves with constant speed $\frac{3}{2}\sqrt{3ag}$ in a horizontal circle with centre at a distance $12a$ below O. The string is inclined at an angle θ to the downward vertical through O.

(a) Find, in terms of a, the extension of the string. [5]

(b) Find the value of k. [3]

6 A particle of mass m kg falls vertically under gravity, from rest. At time t s, P has fallen x m and has velocity $v\,\mathrm{m\,s^{-1}}$. The only forces acting on P are its weight and a resistance of magnitude $kmgv$ N, where k is a constant.

(a) Find an expression for v in terms of t, g and k. [5]

(b) Given that $k = 0.05$, find, in metres, how far P has fallen when its speed is $12\,\mathrm{m\,s^{-1}}$. [5]

7 The points O and P are on a horizontal plane, a distance 8 m apart. A ball is thrown from O with speed $u\,\mathrm{m\,s^{-1}}$ at an angle θ above the horizontal, where $\tan\theta = \frac{4}{3}$. At the same instant, a model aircraft is launched with speed $5\,\mathrm{m\,s^{-1}}$ parallel to the horizontal plane from a point 4 m vertically above P. The model aircraft moves in the same vertical plane as the ball and in the same horizontal direction as the ball. The model aircraft moves horizontally with a constant speed of $5\,\mathrm{m\,s^{-1}}$. After T s, the ball and the model aircraft collide.

(a) Find the value of T. [6]

(b) Find the direction in which the ball is moving immediately before the collision. [3]

Additional page

If you use the following page to complete the answer to any question, the question number must be clearly shown.

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Cambridge Assessment International Education

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FURTHER MATHEMATICS **9231/33**

Paper 3 Further Mechanics 33 **May/June 2023**

MARK SCHEME

Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2023 series for most Cambridge IGCSE, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1: Marks must be awarded in line with: • the specific content of the mark scheme or the generic level descriptors for the question • the specific skills defined in the mark scheme or in the generic level descriptors for the question • the standard of response required by a candidate as exemplified by the standardisation scripts.
GENERIC MARKING PRINCIPLE 2: Marks awarded are always **whole marks** (not half marks, or other fractions).
GENERIC MARKING PRINCIPLE 3: Marks must be awarded **positively**: • marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate • marks are awarded when candidates clearly demonstrate what they know and can do • marks are not deducted for errors • marks are not deducted for omissions • answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.
GENERIC MARKING PRINCIPLE 4: Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.
GENERIC MARKING PRINCIPLE 5: Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).
GENERIC MARKING PRINCIPLE 6: Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Question	Answer	Marks	Guidance
1	$\frac{1}{2}m.3ag - \frac{1}{2}mv^2 = mga(\cos\alpha + \cos\theta)$	M1	Energy equation, 4 terms, dimensionally correct, mass must be present, allow sign errors, allow sin in both terms on RHS
	$mg\cos\theta = \frac{mv^2}{a}$	B1	N2L, may include tension initially but not awarded until tension = 0 used
	$\frac{3}{2}mag - \frac{1}{2}m.ag\cos\theta = mga\left(\frac{4}{5} + \cos\theta\right)$ $\frac{3}{2}\cos\theta = \frac{7}{10}$	M1	Dependent on tension = 0 and on an energy equation, eliminate v^2 .
	$\cos\theta = \frac{7}{15}$	A1	If no *m* in energy equation and no further errors, award SCB2 for correct final answer
		4	

Question	Answer	Marks	Guidance
2	$\frac{1}{2}m\left(v^2-\frac{v^2}{4}\right)=\frac{\frac{1}{2}\lambda mg}{a}\left(\left(\frac{2}{3}a\right)^2-\left(\frac{1}{3}a\right)^2\right)$	M1	Kinetic energy = elastic potential energy, 4 terms, dimensionally correct, allow sign errors.
	$\frac{1}{2}m\left(2ag-\frac{1}{2}ag\right)=\frac{\frac{1}{2}\lambda mg}{a}\left(\left(\frac{2}{3}a\right)^2-\left(\frac{1}{3}a\right)^2\right)$	A1	With v substituted.
	Solve [$\frac{3}{4}v^2=\lambda g\left(\frac{3}{9}a\right)$]	M1	Solve to find value for λ dependent on energy equation with 3 or 4 terms
	$\lambda=\frac{9}{2}$	A1	SCB2 for $\lambda=\frac{9}{2}mg$ if given λ not used
		4	

Question	Answer	Marks	Guidance
3(a)	[Mass is proportional to area] | | Area | Centre of mass from AC | | ABC | $\frac{1}{2}.6a.8a\ (=24a^2)$ | $2a$ | | DEC | $\frac{1}{2}x.5a$ | $\frac{1}{3}x$ | | $ADEB$ | $24a^2-\frac{5}{2}xa$ | $\bar{x}$ |	B1	All correct for ABC and DEC.
	Moments [about AC] $\bar{x}\left(24a^2-\frac{5}{2}xa\right)=24a^2\times 2a-\frac{1}{3}x\times\frac{5}{2}ax$	M1	All moment terms present, dimensionally correct, allow sign error.
		A1	All correct moments about AC.
	$\bar{x}=\dfrac{288a^2-5x^2}{3(48a-5x)}$	A1	AEF
		4	

Question	Answer	Marks	Guidance
3(b)	On the point of toppling about *E*: $\bar{x} = x$, $\dfrac{288a^2 - 5x^2}{3(48a - 5x)} = x$	**B1 FT**	FT *their* expression for $\bar{x}$ from part **(a)**.
	Rearrange to 3-term quadratic: $10x^2 - 144ax + 288a^2 = 0$	**M1**	Allow 3-term inequality.
	$2(5x - 12a)(x - 12a) = 0$, $x = \dfrac{12}{5}a$	**A1**	Single correct answer, no inequality, CWO.
		3	

Question	Answer	Marks	Guidance
4(a)	Let speeds of A and B along line of centres after collision be V_A and V_B $V_A + V_B = u\cos 30°$ (1)	M1	Allow sign errors, allow missing m.
	$-V_A + V_B = eu\cos 30°$ (2)	M1	Signs on LHS must be consistent with (1).
	Speeds perpendicular to line of centres after collision are $u\sin 30°$ and $2u$ Moving in same direction, so $\dfrac{V_A}{u\sin 30°} = \dfrac{V_B}{2u}$ (3)	B1	SOI $V_B = 4V_A$
	Use $V_B = 4V_A$ in (1): $5V_A = u\cos 30°$ From (2): $3V_A = eu\cos 30°$ then Combine to find equation in e only.	M1	A complete method to find equation in e only
	$e = \frac{3}{5}$	A1	

Question	Answer	Marks	Guidance
4(a)	**Alternative method for question 4(a)**		
	Let speeds of A and B along line of centres after collision be V_A and V_B $V_A + V_B = u\cos 30°$ (1)	M1	Allow sign errors, allow missing m.
	$-V_A + V_B = eu\cos 30°$ (2)	M1	Signs on LHS must be consistent with (1).
	Speeds perpendicular to line of centres after collision are $u\sin 30°$ and $2u$ Moving in same direction, so $\dfrac{V_A}{u\sin 30°} = \dfrac{V_B}{2u}$ (3)	B1	SOI $V_B = 4V_A$
	Solve (1) and (2): $V_A = \frac{1}{2}u(1-e)\cos 30°$, $V_B = \frac{1}{2}u(1+e)\cos 30°$ Substitute in (3) to find equation in e only .	M1	Note: $V_A = \dfrac{u}{10}\sqrt{3}$, $V_B = \dfrac{4u}{10}\sqrt{3}$
	$e = \frac{3}{5}$	A1	
		5	
4(b)	KE after $= \frac{1}{2}m\left(V_A^2 + \left(\frac{u}{2}\right)^2\right) + \frac{1}{2}m\left((2u)^2 + V_B^2\right)$	B1	Correct expression for KE for one of the spheres, after collision, with both components.
	KE for A after $= \frac{7}{50}mu^2$ or KE for B after $= \frac{56}{25}mu^2$ or KE loss for A $= \frac{9}{25}mu^2$ or KE **gain** for B $= \frac{6}{25}mu^2$	B1	Implied by total KE after $= \frac{119}{50}mu^2$.
	Total loss in KE $= \frac{3}{25}mu^2$	B1	Term $\frac{1}{2}m(2u)^2$ may be omitted from KE of B before and after.
		3	

Question	Answer	Marks	Guidance
5(a)	$\uparrow T\cos\theta = mg$	**B1**	
	$\rightarrow T\sin\theta = \left[\dfrac{mv^2}{r} = \right] m \times \dfrac{27ag}{4r}$	**B1**	
	$r = 12a\tan\theta$ used	**M1**	
	Divide: $\tan\theta = \dfrac{27}{4\times 12\tan\theta}$, so $\tan\theta = \dfrac{3}{4}$	**M1**	Finds value for $\tan\theta$ OE. Reduces to equation in θ or x, no k.
	$r = 9a$, extension of string $= 3a$	**A1**	
	Alternative method for question 5(a)		
	Let L be stretched length of string. $\uparrow T\cos\theta = mg$	**B1**	Or $T \times \dfrac{12a}{L} = mg$
	$\rightarrow T\sin\theta = \left[\dfrac{mv^2}{r} = \right] m \times \dfrac{27ag}{4r}$	**B1**	
	$r = L\sin\theta$ used	**M1**	
	Use $\cos\theta = \dfrac{12a}{L}$ and $\sin\theta = \dfrac{\left(L^2 - 144a^2\right)^{0.5}}{L}$ and eliminate T.	**M1**	
	$[L^2 - 144a^2 = 81a^2]$ $L = 15a$, extension of string $= 3a$	**A1**	
		5	

Question	Answer	Marks	Guidance
5(b)	Hooke's law: $T = \dfrac{kmg\left(L-12a\right)}{12a}$	**B1**	
	Eliminate T: $\dfrac{kmg\left(L-12a\right)}{12a} = \dfrac{mgL}{12a}$	**M1**	
	$k = \dfrac{L}{L-12a} = 5$	**A1**	
		3	

Question	Answer	Marks	Guidance
6(a)	$m\dfrac{dv}{dt} = mg\left(1-kv\right)$	**B1**	Mass must be seen at this point or earlier. [SUVAT does **not** apply.]
	$-\dfrac{1}{k}\ln\left(1-kv\right) = gt + A$	**M1**	Separate variables and integrate to logarithm.
		A1	Correct, with constant of integration.
	$t = 0, v = 0$ $[A = 0]$	**M1**	Use initial condition to evaluate their constant.
	$v = \dfrac{1}{k}\left(1-e^{-kgt}\right)$	**A1**	Any correct form with v as subject. Final A0 if numerical value of g present.
		5	

Question	Answer	Marks	Guidance
6(b)	$k = 0.05$ and so $\dfrac{dx}{dt} = 20\left(1 - e^{-0.5t}\right)$	*M1	Attempt to integrate if expression contains a term of the form be^{ct}.
	Integrate: $x = 20\left(t + 2e^{-0.5t}\right) + B$	A1	$x = \dfrac{1}{k}\left(t + \dfrac{1}{gk}e^{-kgt}\right) + B$
	$t = 0,\ x = 0\ [B = -40]$	DM1	Use initial condition to evaluate their constant.
	When $v = 12$, from part **(a)**, $e^{-0.5t} = 1 - 0.05 \times 12 = 0.4$, $t = -2\ln 0.4$	M1	1.83…
	$x = -40\ln 0.4 + 40 \times 0.4 - 40 = 12.7$	A1	$40\ln\dfrac{5}{2} - 24$
	Alternative method for question 6(b)		
	$v\dfrac{dv}{dx} = g(1 - kv)$ leading to $\left(1 - \dfrac{1}{1 - kv}\right)dv = -kgdx$	*M1	Separate variables and write in integrable form
	$v + \dfrac{1}{k}\ln(1 - kv) = -kgx + B$	DM1 A1	Dependent on previous M1. Attempt to integrate.
	$v = 0,\ x = 0\ [B = 0]$ and $k = 0.05,\ v = 12\ \ 12 + 20\ln 0.4 = -0.5x$	M1	Dependent on both previous M1s. Use initial condition to evaluate their constant and use $v = 12$
	$x = 12.7$	A1	$40\ln\dfrac{5}{2} - 24$
		5	

Question	Answer	Marks	Guidance
7(a)	For aircraft, $d = 5T$	B1	
	For ball, $\uparrow 4 = u\sin\theta\, T - \frac{1}{2}\times 10\times T^2$	B1	To point of collision.
	For ball, $\rightarrow u\cos\theta\, T = d + 8 = 5T + 8$	B1	
	Eliminate u: $4 = \frac{u4}{5}T - \frac{1}{2}\times 10\times T^2$, $u = \frac{5(4+5T^2)}{4T}$ and $u = \frac{5(5T+8)}{3T}$ $3(4+5T^2) = 4(5T+8)$	*M1	Dependent on LHS of second B1 being 4, expression involving only T
	$3T^2 - 4T - 4 = 0$	DM1	Dependent on previous M1. Obtain and solve 3-term quadratic.
	$T = 2$	A1	Single correct answer.
		6	Note $d - 8$ used leads to $T = \frac{2}{3}$ B1B1B0M1M1A0

Question	Answer	Marks	Guidance
7(b)	$\tan^{-1}\dfrac{(u\sin\theta - 10T)}{u\cos\theta}$	**M1**	OE Accept ‘tan = ……’
	$\tan^{-1}\dfrac{8}{9}$	**A1**	OE
	Direction is 41.6° below the horizontal	**A1**	CAO Note: $d-8$ used leads to 20.9° above the horizontal.
		3	

Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER | CANDIDATE NUMBER

2064332093

FURTHER MATHEMATICS **9231/31**

Paper 3 Further Mechanics **October/November 2022**

1 hour 30 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS
- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.
- Where a numerical value for the acceleration due to gravity (g) is needed, use $10\,\mathrm{m\,s^{-2}}$.

INFORMATION
- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Any blank pages are indicated.

DC (LK/SG) 302027/2

1 A particle of mass 2 kg is attached to one end of a light inextensible string of length 0.6 m. The other end of the string is attached to a fixed point on a smooth horizontal surface. The particle is moving in a circular path on the surface. The tension in the string is 20 N.

Find how many revolutions the particle makes per minute. [3]

2 A light elastic string has natural length a and modulus of elasticity $4mg$. One end of the string is fixed to a point O on a smooth horizontal surface. A particle P of mass m is attached to the other end of the string. The particle P is projected along the surface in the direction OP. When the length of the string is $\frac{5}{4}a$, the speed of P is v. When the length of the string is $\frac{3}{2}a$, the speed of P is $\frac{1}{2}v$.

(a) Find an expression for v in terms of a and g. [4]

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(b) Find, in terms of g, the acceleration of P when the stretched length of the string is $\frac{3}{2}a$. [2]

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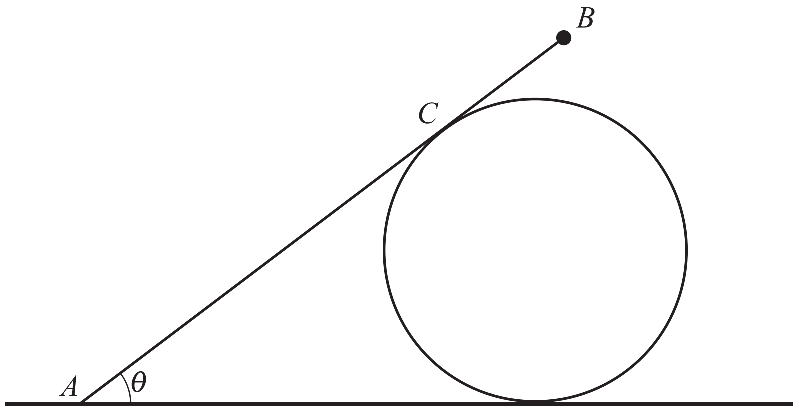

A smooth cylinder is fixed to a rough horizontal surface with its axis of symmetry horizontal. A uniform rod AB, of length $4a$ and weight W, rests against the surface of the cylinder. The end A of the rod is in contact with the horizontal surface. The vertical plane containing the rod AB is perpendicular to the axis of the cylinder. The point of contact between the rod and the cylinder is C, where $AC = 3a$. The angle between the rod and the horizontal surface is θ where $\tan\theta = \frac{3}{4}$ (see diagram). The coefficient of friction between the rod and the horizontal surface is $\frac{6}{7}$.

A particle of weight kW is attached to the rod at B. The rod is about to slip. The normal reaction between the rod and the cylinder is N.

(a) Show that $N = \frac{8}{15}W(1 + 2k)$. [2]

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(b) Find the value of k. [5]

4 A particle of mass 0.5 kg moves along a horizontal straight line. Its velocity is $v\,\mathrm{m\,s^{-1}}$ at time t s. The forces acting on the particle are a driving force of magnitude 50 N and a resistance of magnitude $2v^2$ N. The initial velocity of the particle is $3\,\mathrm{m\,s^{-1}}$.

(a) Find an expression for v in terms of t. [7]

(b) Deduce the limiting value of v. [1]

5 A particle P of mass m is attached to one end of a light inextensible string of length a. The other end of the string is attached to a fixed point O. The string is held taut with OP horizontal. The particle P is projected vertically downwards with speed $\sqrt{\frac{1}{3}ag}$ and starts to move in a vertical circle. P passes through the lowest point of the circle and reaches the point Q where OQ makes an angle θ with the downward vertical. At Q the speed of P is $\sqrt{kag}$ and the tension in the string is $\frac{11}{6}mg$.

(a) Find the value of k and the value of $\cos\theta$. [4]

At Q the particle P becomes detached from the string.

(b) In the subsequent motion, find the greatest height reached by P above the level of the lowest point of the circle. [4]

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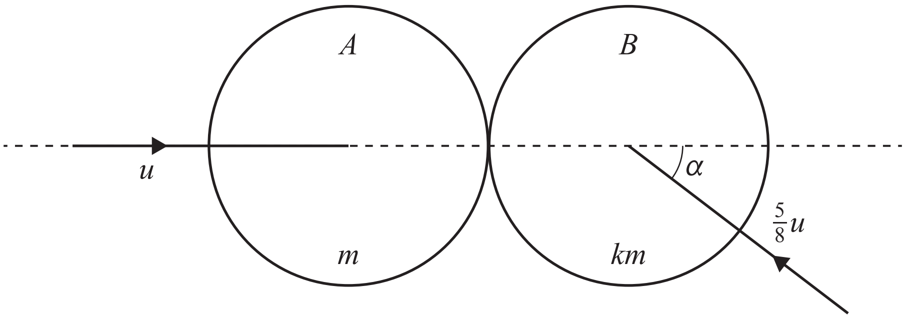

Two uniform smooth spheres A and B of equal radii have masses m and km respectively. The two spheres are moving on a horizontal surface with speeds u and $\frac{5}{8}u$ respectively. Immediately before the spheres collide, A is travelling along the line of centres, and B's direction of motion makes an angle α with the line of centres (see diagram). The coefficient of restitution between the spheres is $\frac{2}{3}$ and $\tan\alpha = \frac{3}{4}$.

After the collision, the direction of motion of B is perpendicular to the line of centres.

(a) Find the value of k. [4]

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(b) Find the loss in the total kinetic energy as a result of the collision. [4]

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7 A particle P is projected with speed $V\,\mathrm{m\,s^{-1}}$ at an angle 75° above the horizontal from a point O on a horizontal plane. It then moves freely under gravity.

(a) Show that the total time of flight, in seconds, is $\frac{2V}{g}\sin 75°$. [2]

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A smooth vertical barrier is now inserted with its lower end on the plane at a distance 15 m from O. The particle is projected as before but now strikes the barrier, rebounds and returns to O. The coefficient of restitution between the barrier and the particle is $\frac{3}{5}$.

(b) Explain why the total time of flight is unchanged. [1]

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(c) Find an expression for V in terms of g. [7]

Additional page

If you use the following page to complete the answer to any question, the question number must be clearly shown.

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Cambridge Assessment International Education

Cambridge International AS & A Level

FURTHER MATHEMATICS **9231/31**

Paper 3 Further Mechanics **October/November 2022**

MARK SCHEME

Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

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This document consists of **12** printed pages.

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GENERIC MARKING PRINCIPLE 2:
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WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	Use F = ma: $20 = \frac{2 \times v^2}{0.6}$ OR $20 = 2 \times 0.6\omega^2$	**M1**	
	$v^2 = 6$ OR $\omega^2 = \frac{50}{3}$	**A1**	
	Number of revolutions per min = $\frac{60v}{0.6 \times 2\pi}$ OR $\frac{60\omega}{2\pi}$ so 39(.0) revolutions	**A1 FT**	38.9848….
		3	

Question	Answer	Marks	Guidance
2(a)	Loss in KE = Gain in EPE, so $\frac{1}{2}mv^2 - \frac{1}{2}m\left(\frac{v}{2}\right)^2 = \frac{1}{2} \times \frac{4mg}{a}\left(\left(\frac{1}{2}a\right)^2 - \left(\frac{1}{4}a\right)^2\right)$	**B1**	EPE terms correct.
		M1	All 4 terms and no extras.
	$\frac{3}{4}mv^2 = \frac{4mg}{a} \times \frac{3}{16}a^2$	**M1**	Simplify.
	$v^2 = ag, \quad v = \sqrt{ag}$	**A1**	
		4	
2(b)	Hooke's law: tension = $\frac{4mg}{a} \times \frac{1}{2}a \; (= 2mg)$	**M1**	
	Acceleration = $\frac{2mg}{m} = 2g$	**A1**	Accept -2g.
		2	

Question	Answer	Marks	Guidance
3(a)	Let F and R be friction and normal reaction at A Take moments about A, for rod $N\times 3a = W\times 2a\cos\theta + kW\times 4a\cos\theta$	M1	Correct terms, allow sign errors and cos/sin mix.
	$3N = (2+4k)W\times\frac{4}{5}$ $N = \frac{8}{15}W(1+2k)$	A1	At least one intermediate line of working. AG
		2	
3(b)	$\uparrow N\cos\theta + R = W + kW$	B1	Resolve (to include R) for rod.
	$\rightarrow F = N\sin\theta$ and $F = \frac{6}{7}R$	B1	Both.
	so $R = \frac{28}{75}W(1+2k)$ or $R = \frac{21}{45}W(1+k)$	M1	Find R or N.
	Eliminate to find k	M1	Complete method.
	$k = \frac{1}{3}$	A1	
		5	

Question	Answer	Marks	Guidance
4(a)	$m\frac{dv}{dt}=50-2v^2 \quad \frac{dv}{dt}=4\left(25-v^2\right)$	**B1**	N2L
	$\frac{1}{10}\int\frac{1}{5-v}+\frac{1}{5+v}dv=\int 4dt$	**M1**	Separate variables and use partial fractions.
	$\frac{1}{10}\left(-\ln(5-v)+\ln(5+v)\right)=4t+A$	**M1 A1**	Integrate into log terms. (Note: formula on MF19).
	Use $t=0,v=3$ to give $A=\frac{1}{10}\ln 4$	**M1**	Use initial condition.
	$4t=\frac{1}{10}\ln\frac{5+v}{4(5-v)}$ leading to $\frac{5+v}{20-4v}=\mathrm{e}^{40t}$	**M1**	Rearrange to make v the subject.
	$v=\frac{5\left(4-\mathrm{e}^{-40t}\right)}{4+\mathrm{e}^{-40t}}$	**A1**	
		7	
4(b)	As $t\to\infty,v\to 5$	**B1**	
		1	

Question	Answer	Marks	Guidance
5(a)	$\frac{1}{2}mv^2 - \frac{1}{2}mu^2 = mga\cos\theta$ $\left[kag = \frac{1}{3}ag + 2ag\cos\theta\right]$	**B1**	Energy equation.
	$T - mg\cos\theta = \frac{m}{a}v^2$ So $\frac{11}{6}mg - mg\cos\theta = \frac{m}{a}.kag$, $\frac{11}{6} - \cos\theta = k$	**B1**	N2L at B.
	Solve simultaneously.	**M1**	
	$k = \frac{4}{3}$, $\cos\theta = \frac{1}{2}$	**A1**	Both.
		4	
5(b)	Initial speed $\uparrow = \sqrt{kag}\sin\theta$	**B1**	
	Use $v^2 = u^2 + 2as$: $0 = \left(\sqrt{kag}\sin\theta\right)^2 - 2gs$	**M1**	
	$s = \frac{1}{2}a$	**A1**	
	Height above lowest point = $s + a - a\cos\theta = \frac{1}{2}a + a - \frac{1}{2}a = a$	**A1 FT**	
		4	

Question	Answer	Marks	Guidance
6(a)	Let speed of A after collision be $\rightarrow v_A$ and speed of B perpendicular to line of centres be $\downarrow v$ Along line of centres: $mu - km\frac{5}{8}u\cos\alpha = mv_A$	M1	Momentum.
	NEL: $0 - v_A = e\left(\frac{5}{8}u\ \cos\alpha + u\right)$	M1	NEL
	So $u - \frac{5}{8}ku\cos\alpha = -\frac{2}{3}\left(\frac{5}{8}u\ \cos\alpha + u\right)$	M1	Solve.
	Substitute for cos, to give $k = 4$	A1	
		4	
6(b)	$v_B = \frac{5}{8}u\sin\alpha\ = \frac{3}{8}u$	B1	Velocity perpendicular to line of centres
	$v_A = -u$	B1 FT	
	KE before = $\frac{1}{2}mu^2 + \frac{1}{2}km\left(\frac{5}{8}u\right)^2 = \frac{1}{2}mu^2 + \frac{25}{32}mu^2 = \frac{41}{32}mu^2$ KE after = $\frac{1}{2}mv_A{}^2 + \frac{1}{2}kmv_B{}^2 = \frac{1}{2}mu^2 + 2m\frac{9}{64}u^2 = \frac{25}{32}mu^2$	M1	NOTE: KE before and after for *A* is unchanged. Both.
	Loss = $mu^2\left(\frac{41}{32} - \frac{25}{32}\right) = \frac{1}{2}mu^2$	A1	
		4	

Question	Answer	Marks	Guidance
7(a)	$\uparrow 0 = V\sin 75t - \frac{1}{2}gt^2$	**M1**	
	$t = \frac{2V}{g}\sin 75°$	**A1**	AG
		2	
7(b)	Vertical component of velocity is unchanged.	**B1**	
		1	

Question	Answer	Marks	Guidance
7(c)	Horizontally to wall, $\rightarrow 15 = V\cos 75t;\left(t = \frac{15}{V\cos 75}\right)$	B1	
	eVcos75	B1	Speed after rebound.
	$T = \frac{15}{eV\cos 75}$	M1	Time back to O $\left(t = \frac{3}{5}T\right)$
	Vertically for whole flight: $t + T = \frac{2V}{g}\sin 75°$	M1	
	$\frac{15}{V\cos 75} + \frac{15}{eV\cos 75} = \frac{2V}{g}\sin 75°$	A1	
	$V^2 \cos 75 \sin 75 = 20g$	M1	
	Multiply by 2: $V^2 \sin 150 = 40g$, $V^2 = 80g$ $V = 4\sqrt{5g}\left(= 8.94\sqrt{g}\right)$	A1	
		7	

Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

1076280977

FURTHER MATHEMATICS **9231/32**

Paper 3 Further Mechanics **October/November 2022**

1 hour 30 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.
- Where a numerical value for the acceleration due to gravity (g) is needed, use $10\,\mathrm{m\,s^{-2}}$.

INFORMATION

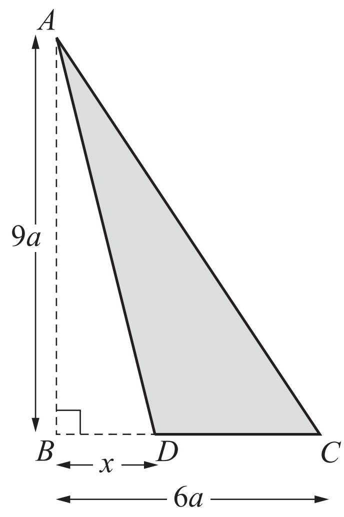
- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Any blank pages are indicated.

1 A particle P of mass m is attached to one end of a light inextensible string of length a. The other end of the string is attached to a fixed point O. The string is held taut with OP making an angle α with the downward vertical, where $\cos\alpha = \frac{2}{3}$. The particle P is projected perpendicular to OP in an upwards direction with speed $\sqrt{3ag}$. It then starts to move along a circular path in a vertical plane.

Find the cosine of the angle between the string and the upward vertical when the string first becomes slack. [4]

2



A uniform lamina is in the form of a triangle ABC in which angle B is a right angle, $AB = 9a$ and $BC = 6a$. The point D is on BC such that $BD = x$ (see diagram). The region ABD is removed from the lamina. The resulting shape ADC is placed with the edge DC on a horizontal surface and the plane ADC is vertical.

Find the set of values of x, in terms of a, for which the shape is in equilibrium. [6]

3 One end of a light elastic string, of natural length a and modulus of elasticity $\frac{16}{3}Mg$, is attached to a fixed point O. A particle P of mass $4M$ is attached to the other end of the string and hangs vertically in equilibrium. Another particle of mass $2M$ is attached to P and the combined particle is then released from rest. The speed of the combined particle when it has descended a distance $\frac{1}{4}a$ is v.

Find an expression for v in terms of g and a. [6]

4 A particle P of mass 5 kg moves along a horizontal straight line. At time t s, the velocity of P is $v\,\mathrm{m\,s^{-1}}$ and its displacement from a fixed point O on the line is x m. The forces acting on P are a force of magnitude $\frac{500}{v}$ N in the direction OP and a resistive force of magnitude $\frac{1}{2}v^2$ N. When $t = 0$, $x = 0$ and $v = 5$.

(a) Find an expression for v in terms of x. [6]

(b) State the value that the speed approaches for large values of x. [1]

5 A particle P is projected with speed $u\,\mathrm{m\,s^{-1}}$ at an angle of θ above the horizontal from a point O on a horizontal plane and moves freely under gravity. The horizontal and vertical displacements of P from O at a subsequent time t s are denoted by x m and y m respectively.

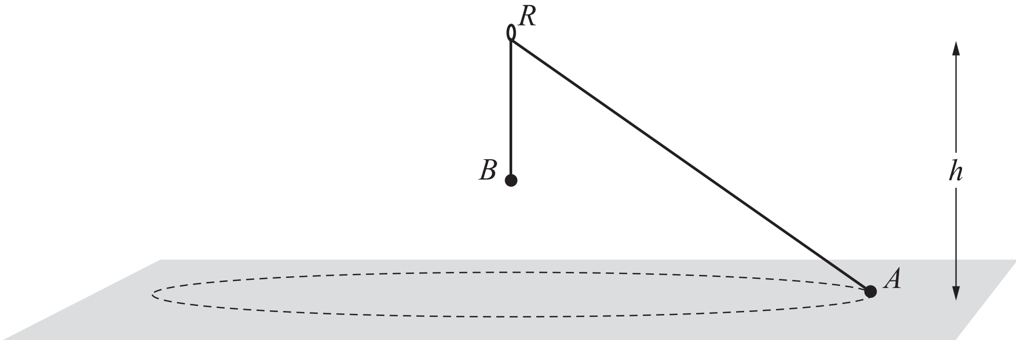
(a) Show that the equation of the trajectory is given by

$$y = x\tan\theta - \frac{gx^2}{2u^2}(1+\tan^2\theta).$$ [4]

In the subsequent motion P passes through the point with coordinates (30, 20).

(b) Given that one possible value of $\tan\theta$ is $\frac{4}{3}$, find the other possible value of $\tan\theta$. [5]

6



A light inextensible string is threaded through a fixed smooth ring R which is at a height h above a smooth horizontal surface. One end of the string is attached to a particle A of mass m. The other end of the string is attached to a particle B of mass $\frac{6}{7}m$. The particle A moves in a horizontal circle on the surface. The particle B hangs in equilibrium below the ring and above the surface (see diagram).

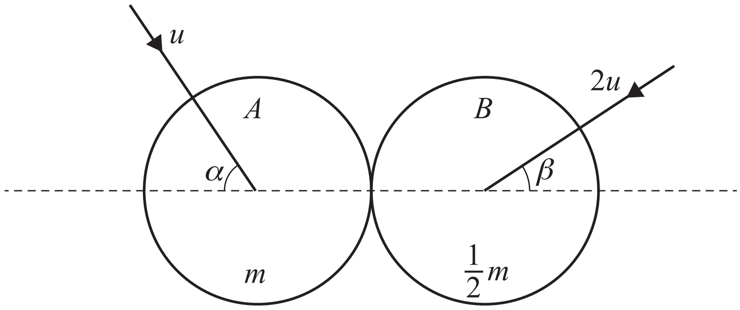
When A has constant angular speed ω, the angle between AR and BR is θ and the normal reaction between A and the surface is N.

When A has constant angular speed $\frac{3}{2}\omega$, the angle between AR and BR is α and the normal reaction between A and the surface is $\frac{1}{2}N$.

(a) Show that $\cos\theta = \frac{4}{9}\cos\alpha$. [5]

(b) Find N in terms of m and g and find the value of $\cos\alpha$. [4]

7



Two uniform smooth spheres A and B of equal radii have masses m and $\frac{1}{2}m$ respectively. The two spheres are moving on a horizontal surface when they collide. Immediately before the collision, sphere A is travelling with speed u and its direction of motion makes an angle α with the line of centres. Sphere B is travelling with speed $2u$ and its direction of motion makes an angle β with the line of centres (see diagram). The coefficient of restitution between the spheres is $\frac{5}{8}$ and $\alpha + \beta = 90°$.

(a) Find the component of the velocity of B parallel to the line of centres after the collision, giving your answer in terms of u and α. [4]

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The direction of motion of B after the collision is parallel to the direction of motion of A before the collision.

(b) Find the value of $\tan\alpha$. [5]

Additional page

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Question	Answer	Marks	Guidance
1	When string goes slack, $mg\cos\beta = \frac{m}{a}v^2$, $v^2 = ag\cos\beta$	B1	N2L May include T, but B1 not awarded until $T = 0$.
	$\frac{1}{2}m.3ag - \frac{1}{2}mv^2 = mg\left(a\cos\alpha + a\cos\beta\right)$	B1	Energy equation.
	So $u^2 - ag\cos\beta = 2ag\left(\cos\beta + \frac{2}{3}\right)$	M1	Combine.
	$\cos\beta = \frac{u^2 - \frac{4}{3}ag}{3ag} = \frac{5}{9}$	A1	
		4	

<table>
<tr><th>Question</th><th>Answer</th><th>Marks</th><th>Guidance</th></tr>
<tr><td>2</td><td>

	Area	Distance from AB
ABC	$27a^2$	$2a$
ABD	$\frac{9}{2}ax$	$\frac{1}{3}x$
Shape *ADC*	$27a^2 - \frac{9}{2}ax$	$\bar{x}$

</td><td></td><td></td></tr>
<tr><td></td><td>Taking moments about AB
$\bar{x} \times \left(27a^2 - \frac{9}{2}ax\right) = 27a^2 \times 2a - \frac{9}{2}ax \times \frac{1}{3}x$
$\left[\bar{x} = \frac{54a^3 - \frac{3}{2}ax^2}{27a^2 - \frac{9}{2}ax}\right]$</td><td>M1
A1
A1</td><td>Moments equation with 3 terms.
At least 2 terms correct.
All correct.</td></tr>
<tr><td></td><td>For equilibrium, $x \leqslant \bar{x}$,
$54a^3 - \frac{3}{2}ax^2 \geqslant x\left(27a^2 - \frac{9}{2}ax\right)$</td><td>B1</td><td>Use correct condition: allow strict inequality.
Can be implied by correct final answer $x \leqslant 3a$.</td></tr>
<tr><td></td><td>$54a^2 - 27ax + 3x^2 \geqslant 0$
$(x-3a)(x-6a) \geqslant 0$</td><td>M1</td><td>Simplify and attempt to solve a quadratic inequality or equation.</td></tr>
<tr><td></td><td>$(0 \leqslant)\ x \leqslant 3a$ [only]</td><td>A1</td><td>CAO</td></tr>
</table>

Question	Answer	Marks	Guidance
2	**Alternative method for question 2**		
	Taking moments with B as origin.	**M1**	
	$\bar{x} = \frac{1}{3}(0 + x + 6a) = 2a + \frac{1}{3}x$	**A2**	
	For equilibrium, $x \leqslant \bar{x}$, so $x \leqslant 2a + \frac{1}{3}x$	**B1**	Allow strict inequality.
	$(0 \leqslant)\ x \leqslant 3a$	**M1**	
		A1	
		6	

Question	Answer	Marks	Guidance
3	In equilibrium, $\dfrac{\frac{16}{3}Mge}{a} = 4Mg, \quad e = \dfrac{3}{4}a$	**B1**	
	In subsequent motion, Loss in GPE = gain in EPE + gain in KE	**M1**	Energy equation with GPE and KE terms correct and at least one EPE term. Dimensionally correct.
	$\dfrac{6Mga}{4} = \dfrac{1}{2}.\dfrac{16}{3}.\dfrac{Mg}{a}.\left(a^2 - \left(\dfrac{3a}{4}\right)^2\right) + \dfrac{1}{2}.6Mv^2$	**B1**	EPE correct.
		A1	All correct.
	$\dfrac{3Mga}{2} = \dfrac{8Mg}{3a}.\dfrac{7}{16}a^2 + 3Mv^2$ etc	**M1**	Attempt to find v in terms of a and g.
	$\dfrac{ga}{3} = 3v^2, \quad v = \dfrac{1}{3}\sqrt{ga}$	**A1**	
		6	

Question	Answer	Marks	Guidance
4(a)	$5v\frac{dv}{dx}=\frac{500}{v}-\frac{1}{2}v^2$	B1	Sight of m or 5 is required.
	$\frac{10v^2dv}{1000-v^3}=dx$	M1	Separate variables and attempt to integrate into a log term.
	$-\frac{10}{3}\ln\left(1000-v^3\right)=x(+A)$	A1	
	$x=0, v=5, \quad A=-\frac{10}{3}\ln 875$	M1	Evaluate constant: correct initial condition used.
	$x=\frac{10}{3}\ln\frac{875}{1000-v^3}$	M1	Make v the subject: correct use of logs.
	$v=\left[(1000-875e^{-0.3x}\right]^{\frac{1}{3}}$	A1	$v=5\left[(8-7e^{-0.3x}\right]^{\frac{1}{3}}$: A0 if $e^{\ln}$ terms.
		6	
4(b)	Maximum value of v is 10	B1	No FT: result can be found from initial equation.
		1	

Question	Answer	Marks	Guidance
5(a)	$\rightarrow x = u\cos\theta t$	B1	Result quoted from MF19 scores 0/4.
	$\uparrow y = u\sin\theta t - \frac{1}{2}gt^2$	B1	
	Eliminate t: $y = u\sin\theta \times \frac{x}{\cos\theta} - \frac{1}{2}g\left(\frac{x}{u\cos\theta}\right)^2$	M1	
	$y = x\tan\theta - \frac{gx^2}{2u^2}(1+\tan^2\theta)$	A1	Must be an intermediate line of working. AG
		4	
5(b)	$20 = 30\times\frac{4}{3} - 10\times\frac{30^2}{2u^2}\times\left(1+\left(\frac{4}{3}\right)^2\right)$	M1	Substituting values correctly.
	$u^2 = 625$, $[u = 25]$	A1	
	Substitute back into trajectory equation, $20 = 30\tan\theta - \frac{g30^2}{2.25^2}\sec^2\theta = 30\tan\theta - \frac{36}{5}(1+\tan^2\theta)$ $18\tan^2\theta - 75\tan\theta + 68 = 0$	M1	Obtain a 3-term quadratic.
	One solution is $\frac{4}{3}$, $(3\tan\theta - 4)(6\tan\theta - 17) = 0$	M1	
	Giving $\tan\theta = \frac{17}{6}$	A1	
		5	

Question	Answer	Marks	Guidance
6(a)	$T = \frac{6}{7}mg$	**B1**	May be implied.
	$T\sin\theta = mr\omega^2 = mh\tan\theta \times \omega^2$	**B1**	Allow *r* for radius.
	Radius of circle = $h\tan\theta$ [So $\omega^2 = \frac{6g}{7h}\cos\theta$]	**B1**	
	In second scenario, $\frac{9}{4}\omega^2 = \frac{6g}{7h}\cos\alpha$	**M1**	Second scenario, equivalent result .
	Equate, $\frac{6g}{7h}\cos\theta = \frac{4}{9}\times\frac{6g}{7h}\cos\alpha$ giving $\cos\theta = \frac{4}{9}\cos\alpha$	**A1**	Combine convincingly to obtain given result. AG
		5	

Question	Answer	Marks	Guidance
6(b)	First scenario: $N + T\cos\theta = mg$		
	Second scenario, $\frac{1}{2}N + T\cos\alpha = mg$	**B1**	Both.
	Equate: $mg - \frac{6}{7}mg\cos\theta = 2mg - \frac{12}{7}mg\cos\alpha$	**M1**	$12\cos\alpha - 6\cos\theta = 7$
	$\cos\alpha = \frac{3}{4}$	**A1**	
	$N = \frac{5}{7}mg$	**A1**	
		4	

Question	Answer	Marks	Guidance
7(a)	Let v, w be speeds of A and B along line of centres after collision $mv + \frac{1}{2}mw = mu\cos\alpha - \frac{1}{2}m.2u\cos\beta$	M1	Momentum: masses correct, opposite signs on RHS.
	$w - v = e(2u\cos\beta + u\cos\alpha)$	M1	NEL: LHS signs must be consistent with momentum equation, same sign for both terms on RHS.
	$\alpha + \beta = 90°$, so $\cos\beta = \sin\alpha$ Use this fact and solve to find w	M1	Solve to find an expression of the correct form.
	$w = \frac{2}{3}u\left(\frac{1}{4}\sin\alpha + \frac{13}{8}\cos\alpha\right)$	A1	
		4	

Question	Answer	Marks	Guidance
7(b)	Perpendicular to line of centres, speed of B is $2u\sin\beta = 2u\cos\alpha$	**B1**	
	After, velocity of B makes angle α with line of centres, so $\tan\alpha = \dfrac{2u\cos\alpha}{w}$	**B1**	
	$\dfrac{\sin\alpha}{\cos\alpha} = \dfrac{2u\cos\alpha}{\frac{2}{3}u\left(\frac{1}{4}\sin\alpha + \frac{13}{8}\cos\alpha\right)}$ giving	**M1***	Obtain homogeneous equation in cos and sin or an equation in tan
	$3(\cos\alpha)^2 = \frac{1}{4}(\sin\alpha)^2 + \frac{13}{8}\sin\alpha\cos\alpha$ $2(\tan\alpha)^2 + 13\tan\alpha - 24 = 0$, $(2\tan\alpha - 3)(\tan\alpha + 8) = 0$	**DM1**	Obtain quadratic and solve to find values of $\tan\alpha$
	$\tan\alpha = \dfrac{3}{2}$	**A1**	
		5	

Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

5661734196

FURTHER MATHEMATICS **9231/32**

Paper 3 Further Mechanics **May/June 2022**

1 hour 30 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.
- Where a numerical value for the acceleration due to gravity (g) is needed, use $10\,\text{m}\,\text{s}^{-2}$.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

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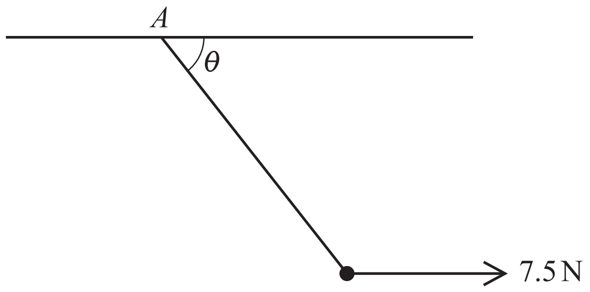

A particle of weight 10 N is attached to one end of a light elastic string. The other end of the string is attached to a fixed point A on a horizontal ceiling. A horizontal force of 7.5 N acts on the particle. In the equilibrium position, the string makes an angle θ with the ceiling (see diagram). The string has natural length 0.8 m and modulus of elasticity 50 N.

(a) Find the tension in the string. [2]

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(b) Find the vertical distance between the particle and the ceiling. [3]

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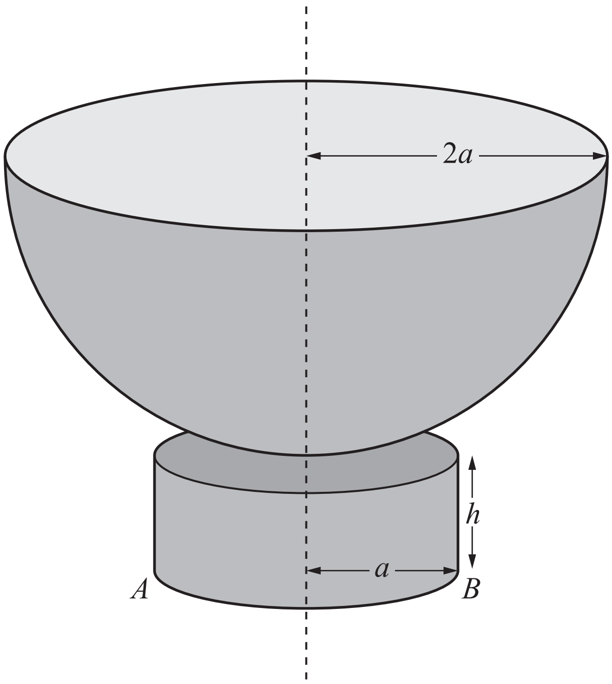
2 One end of a light inextensible string of length a is attached to a fixed point O. A particle of mass m is attached to the other end of the string. The particle is held at the point A with the string taut. The angle between OA and the downward vertical is equal to α, where $\cos\alpha = \frac{4}{5}$. The particle is projected from A, perpendicular to the string in an upwards direction, with a speed $\sqrt{3ga}$. It then moves along a circular path in a vertical plane. The string first goes slack when it makes an angle θ with the upward vertical through O.

Find the value of $\cos\theta$. [5]

3 A particle P is moving in a horizontal straight line. Initially P is at the point O on the line and is moving with velocity $25\,\mathrm{m\,s^{-1}}$. At time t s after passing through O, the acceleration of P is $\frac{4000}{(5t+4)^3}\,\mathrm{m\,s^{-2}}$ in the direction PO. The displacement of P from O at time t is x m.

Find an expression for x in terms of t. [5]

4



An object is composed of a hemispherical shell of radius $2a$ attached to a closed hollow circular cylinder of height h and base radius a. The hemispherical shell and the hollow cylinder are made of the same uniform material. The axes of symmetry of the shell and the cylinder coincide. AB is a diameter of the lower end of the cylinder (see diagram).

(a) Find, in terms of a and h, an expression for the distance of the centre of mass of the object from AB. [4]

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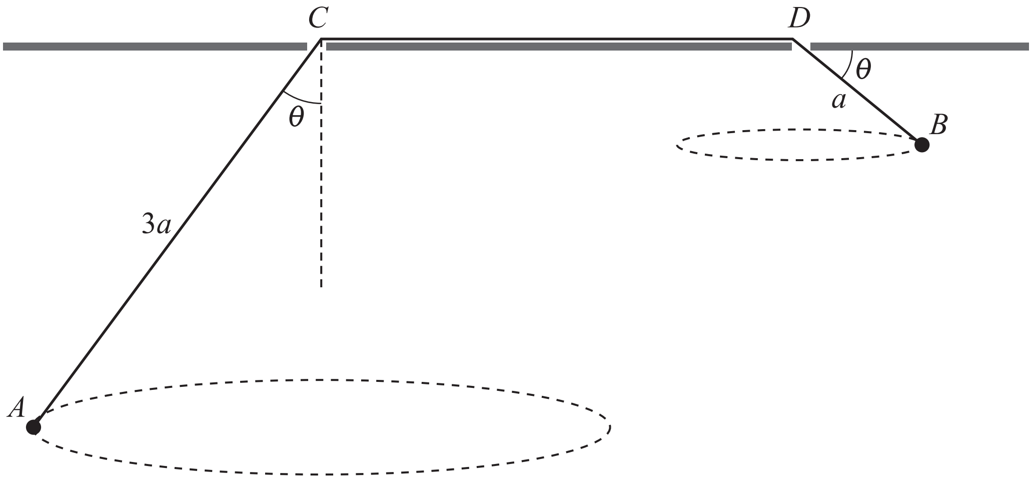
The object is placed on a rough plane which is inclined to the horizontal at an angle θ, where $\tan\theta = \frac{2}{3}$. The object is in equilibrium with AB in contact with the plane and lying along a line of greatest slope of the plane.

(b) Find the set of possible values of h, in terms of a. [4]

5



A light inextensible string AB passes through two small holes C and D in a smooth horizontal table where $AC = 3a$ and $DB = a$. A particle of mass m is attached at the end A and moves in a horizontal circle with angular velocity ω. A particle of mass $\frac{3}{4}m$ is attached to the end B and moves in a horizontal circle with angular velocity $k\omega$. AC makes an angle θ with the downward vertical and DB makes an angle θ with the horizontal (see diagram).

Find the value of k. [7]

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6 Two uniform smooth spheres A and B of equal radii have masses m and km respectively. The two spheres are on a horizontal surface. Sphere A is travelling with speed u towards sphere B which is at rest. The spheres collide. Immediately before the collision, the direction of motion of A makes an angle α with the line of centres. The coefficient of restitution between the spheres is $\frac{1}{2}$.

(a) Show that the speed of B after the collision is $\dfrac{3u\cos\alpha}{2(1+k)}$ and find also an expression for the speed of A along the line of centres after the collision, in terms of k, u and α. [4]

After the collision, the kinetic energy of A is equal to the kinetic energy of B.

(b) Given that $\tan\alpha = \frac{2}{3}$, find the possible values of k. [5]

7 Particles P and Q are projected in the same vertical plane from a point O at the top of a cliff. The height of the cliff exceeds 50 m. Both particles move freely under gravity. Particle P is projected with speed $\frac{35}{2}\,\mathrm{m\,s^{-1}}$ at an angle α above the horizontal, where $\tan\alpha = \frac{4}{3}$. Particle Q is projected with speed $u\,\mathrm{m\,s^{-1}}$ at an angle β above the horizontal, where $\tan\beta = \frac{1}{2}$. Particle Q is projected one second after the projection of particle P. The particles collide T s after the projection of particle Q.

(a) Write down expressions, in terms of T, for the horizontal displacements of P and Q from O when they collide and hence show that $4uT = 21\sqrt{5}(T+1)$. [4]

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(b) Find the value of T. [4]

(c) Find the horizontal and vertical displacements of the particles from O when they collide. [3]

Additional page

If you use the following page to complete the answer to any question, the question number must be clearly shown.

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Cambridge Assessment International Education

Cambridge International AS & A Level

FURTHER MATHEMATICS **9231/32**

Paper 3 Further Mechanics **May/June 2022**

MARK SCHEME

Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2022 series for most Cambridge IGCSE, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

This document consists of **14** printed pages.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1: Marks must be awarded in line with: • the specific content of the mark scheme or the generic level descriptors for the question • the specific skills defined in the mark scheme or in the generic level descriptors for the question • the standard of response required by a candidate as exemplified by the standardisation scripts.
GENERIC MARKING PRINCIPLE 2: Marks awarded are always **whole marks** (not half marks, or other fractions).
GENERIC MARKING PRINCIPLE 3: Marks must be awarded **positively**: • marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate • marks are awarded when candidates clearly demonstrate what they know and can do • marks are not deducted for errors • marks are not deducted for omissions • answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.
GENERIC MARKING PRINCIPLE 4: Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.
GENERIC MARKING PRINCIPLE 5: Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).
GENERIC MARKING PRINCIPLE 6: Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1(a)	$T\cos\theta = 7.5, \quad T\sin\theta = 10$	B1	
	$T = \left(7.5^2 + 10^2\right)^{\frac{1}{2}} = 12.5$ N	B1	
		2	
1(b)	Hooke's law: $T = \dfrac{50x}{0.8}$, $x = 0.2$	B1	
	$(x + 0.8)\sin\theta = 1 \times \dfrac{10}{12.5}$	M1	
	Vertical distance $= \dfrac{4}{5} = 0.8$	A1	
		3	

Question	Answer	Marks	Guidance
2	$-\dfrac{1}{2}mv^2 + \dfrac{1}{2}m \times 3ga = mga(\cos\theta + \cos\alpha)$	M1 A1	Energy equation.
	$(T+)mg\cos\theta = \dfrac{m}{a}v^2$	B1	N2L
	$ag\cos\theta = 3ga - 2ga\cos\theta - 2ga \times \dfrac{4}{5}$	M1	Combine to find $\cos\theta$.
	$\cos\theta = \dfrac{7}{15}$	A1	
		5	

Question	Answer	Marks	Guidance
3	$\frac{dv}{dt} = -\frac{4000}{(5t+4)^3}$; $v = \frac{400}{(5t+4)^2}(+A)$	M1 A1	Integrate. Constant of integration needed for A1.
	$t = 0, \quad v = 25 \quad A = 25 - 25 = 0$	M1	Find constant.
	$v = \frac{dx}{dt}$: $x = -80(5t+4)^{-1} + B$ $x = 0, \ t = 0 \quad B = 20$	M1	Integrate and find constant.
	$x = \frac{-80}{5t+4} + 20 \quad \left(= \frac{100t}{5t+4}\right)$	A1	
		5	

Question	Answer	Marks	Guidance
4(a)	<table><tr><td></td><td>Area</td><td>Centre of mass from *AB*</td></tr><tr><td>Cylinder</td><td>$2\pi ah + 2\pi a^2$</td><td>$\frac{1}{2}h$</td></tr><tr><td>Shell</td><td>$2\pi(2a)^2$</td><td>$h + a$</td></tr></table>	M1	Moments equation, condone missing ends of cylinder. One expression on the RHS correct.
	Moments about *AB* $\bar{x}\left(2\pi ah + 2\pi a^2 + 2\pi(2a)^2\right) = 2\pi(2a)^2 \times (h+a) + \left(2\pi ah + 2\pi a^2\right)\left(\frac{1}{2}h\right)$	A1 A1	One correct expression on RHS correct scores A1.
	$(2h+10a)\bar{x} = h^2 + ah + 8ah + 8a^2$ $\bar{x} = \frac{h^2 + 9ah + 8a^2}{2(h+5a)}$	A1	
		4	
4(b)	$\tan\theta \leqslant \frac{a}{\bar{x}}$	B1	
	$\bar{x} = \frac{h^2 + 9ah + 8a^2}{2(h+5a)} \leqslant \frac{3}{2}a$ $h^2 + 6ah - 7a^2 \leqslant 0$	M1	Form inequality and rearrange to quadratic, condone equation.
	$(h-a)(h+7a) \leqslant 0$	M1	Attempt to solve, condone equation.
	$(-7a \leqslant)\, h \leqslant a$	A1	
		4	

Question	Answer	Marks	Guidance
5	For A: $T\sin\theta = mr\omega^2$	M1	N2L horizontal.
	$r = 3a\sin\theta$	B1	Correct expression for radius.
	$T = m \times 3a\omega^2$	A1	
	Similarly, for B: $T\cos\theta = \frac{3}{4}m \times r \times k^2\omega^2$	M1	N2L horizontal
	$T = \frac{3}{4}mak^2\omega^2$	A1	
	$m \times 3a\omega^2 = \frac{3}{4}mak^2\omega^2$	M1	Equate expressions for T.
	$k^2 = 4, \quad k = 2$	A1	

Question	Answer	Marks	Guidance
5	**Alternative method for question 5**		
	For A: $T\cos\theta = mg$, $T\sin\theta = mr\omega^2$	**M1**	N2L horizontal and vertical.
	$r = 3a\sin\theta$	**B1**	Correct expression for radius.
	$T = m\times 3a\omega^2 = \frac{5}{4}mg$, $\omega^2 = \frac{5}{12}\times\frac{g}{a}$	**A1**	Combine to obtain expression for ω^2.
	Similarly, for B: $T\cos\theta = \frac{3}{4}m\times r\times k^2\omega^2$	**M1**	N2L horizontal.
	$T = \frac{3}{4}mak^2\omega^2$	**A1**	
	$\frac{5}{4}mg = \frac{3}{4}mak^2\times\frac{5g}{12a}$	**M1**	Substitute for T and ω.
	$k^2 = 4$, $k = 2$	**A1**	
		7	

Question	Answer	Marks	Guidance
6(a)	Let v and w be speeds after collision: $mv + kmw = mu\cos\alpha$	**M1**	Momentum along line of centres.
	$w - v = \frac{1}{2}u\cos\alpha$	**M1**	NEL consistent signs.
	Add to give $\frac{3u\cos\alpha}{2(1+k)}$	**A1**	AG Convincing working.
	Substitute back or re-solve: $v = \left\|\frac{(2-k)u\cos\alpha}{2(1+k)}\right\|$	**A1**	Accept without modulus sign.
		4	

Question	Answer	Marks	Guidance
6(b)	$\sqrt{(u\sin\alpha)^2+\left(\frac{(2-k)u\cos\alpha}{2(1+k)}\right)^2}$	B1	For speed of A (SOI).
	Equal KE after collision: $\frac{1}{2}km\left(\frac{3u\cos\alpha}{2(1+k)}\right)^2=\frac{1}{2}m\left((u\sin\alpha)^2+\left(\frac{(2-k)u\cos\alpha}{2(1+k)}\right)^2\right)$ $\left[9k(\cos\alpha)^2=4(1+k)^2(\sin\alpha)^2+(2-k)^2(\cos\alpha)^2\right]$	M1	Equate KEs.
	Use $\tan\alpha=\frac{2}{3}$: $16\left(1+2k+k^2\right)+9\left(4-4k+k^2\right)=81k$	M1	
	$25k^2-85k+52=0$ leading to $(5k-4)(5k-13)=0$	M1	Obtain quadratic and attempt to solve.
	$k=\frac{4}{5}$ or $\frac{13}{5}$	A1	
		5	

Question	Answer	Marks	Guidance
7(a)	For Q: $x = u\cos\beta T$	**B1**	
	For P: $x = \frac{35}{2}\cos\alpha(T+1)$	**B1**	
	Collision, so $\frac{35}{2}\cos\alpha(T+1) = u\cos\beta T$	**M1**	Equate and attempt to rearrange.
	$\frac{35}{2}\times\frac{3}{5}(T+1) = u\times\frac{2}{\sqrt{5}}T$ $4uT = 21\sqrt{5}(T+1)$	**A1**	AG Shown convincingly.
		4	
7(b)	Vertical motion to collision: For Q: $y = u\sin\beta T - \frac{1}{2}gT^2$ For P: $y = \frac{35}{2}\sin\alpha(T+1) - \frac{1}{2}g(T+1)^2$	**M1 A1**	M1 for both expressions, one correct.
	Equate: $u\times\frac{1}{\sqrt{5}}T - \frac{1}{2}gT^2 = \frac{35}{2}\times\frac{4}{5}(T+1) - \frac{1}{2}g(T+1)^2$ $14(T+1) - \frac{1}{2}g\left(T^2+2T+1-T^2\right) = \frac{21}{4}(T+1)$	**M1**	Equate and attempt to solve
	$16T+36 = 21T+21, \quad 15 = 5T$ $T = 3$	**A1**	
		4	

Question	Answer	Marks	Guidance
7(c)	$x = 42$	**B1**	
	$\|y\| = 24$	**M1**	
	$y = -24$ (or 24 m below O)	**A1**	Correct sign or in words.
		3	

Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

7327543339

FURTHER MATHEMATICS **9231/33**

Paper 3 Further Mechanics **May/June 2022**

1 hour 30 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.
- Where a numerical value for the acceleration due to gravity (g) is needed, use $10\,\mathrm{m\,s^{-2}}$.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Any blank pages are indicated.

DC (LK/SG) 219278/1

1 A uniform lamina $OABC$ is a trapezium whose vertices can be represented by coordinates in the x-y plane. The coordinates of the vertices are $O(0,0)$, $A(15,0)$, $B(9,4)$ and $C(3,4)$.

Find the x-coordinate of the centre of mass of the lamina. [4]

2 A particle P of mass m is attached to one end of a light elastic string of natural length a and modulus of elasticity $\frac{4}{3}mg$. The other end of the string is attached to a fixed point O on a rough horizontal surface. The particle is at rest on the surface with the string at its natural length. The coefficient of friction between P and the surface is $\frac{1}{3}$. The particle is projected along the surface in the direction OP with a speed of $\frac{1}{2}\sqrt{ga}$.

Find the greatest extension of the string during the subsequent motion. [5]

3 A particle P is projected with speed $25\,\mathrm{m\,s^{-1}}$ at an angle θ above the horizontal from a point O on a horizontal plane and moves freely under gravity. After $2\,\mathrm{s}$ the speed of P is $15\,\mathrm{m\,s^{-1}}$.

(a) Find the value of $\sin\theta$. [5]

(b) Find the range of the flight. [3]

4 One end of a light inextensible string of length a is attached to a fixed point O. A particle of mass m is attached to the other end of the string and is held with the string taut at the point A. At A the string makes an angle θ with the upward vertical through O. The particle is projected perpendicular to the string in a downward direction from A with a speed u. It moves along a circular path in the vertical plane.

When the string makes an angle α with the downward vertical through O, the speed of the particle is $2u$ and the magnitude of the tension in the string is 10 times its magnitude at A.

It is given that $u = \sqrt{\frac{2}{3}ga}$.

(a) Find, in terms of m and g, the magnitude of the tension in the string at A. [6]

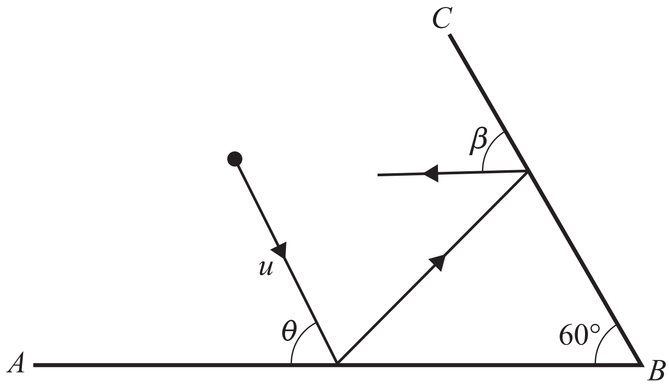
(b) Find the value of $\cos\alpha$. [2]

5 A particle P of mass 4 kg is moving in a horizontal straight line. At time t s the velocity of P is $v\,\mathrm{m\,s^{-1}}$ and the displacement of P from a fixed point O on the line is x m. The only force acting on P is a resistive force of magnitude $(4\mathrm{e}^{-x} + 12)\mathrm{e}^{-x}$ N. When $t = 0$, $x = 0$ and $v = 4$.

(a) Show by integration that $v = \dfrac{1 + 3\mathrm{e}^{x}}{\mathrm{e}^{x}}$. [4]

(b) Find an expression for x in terms of t. [4]

6



AB and BC are two fixed smooth vertical barriers on a smooth horizontal surface, with angle $ABC = 60°$. A particle of mass m is moving with speed u on the surface. The particle strikes AB at an angle θ with AB. It then strikes BC and rebounds at an angle β with BC (see diagram). The coefficient of restitution between the particle and each barrier is e and $\tan\theta = 2$.

The kinetic energy of the particle after the first collision is 40% of its kinetic energy before the first collision.

(a) Find the value of e. [4]

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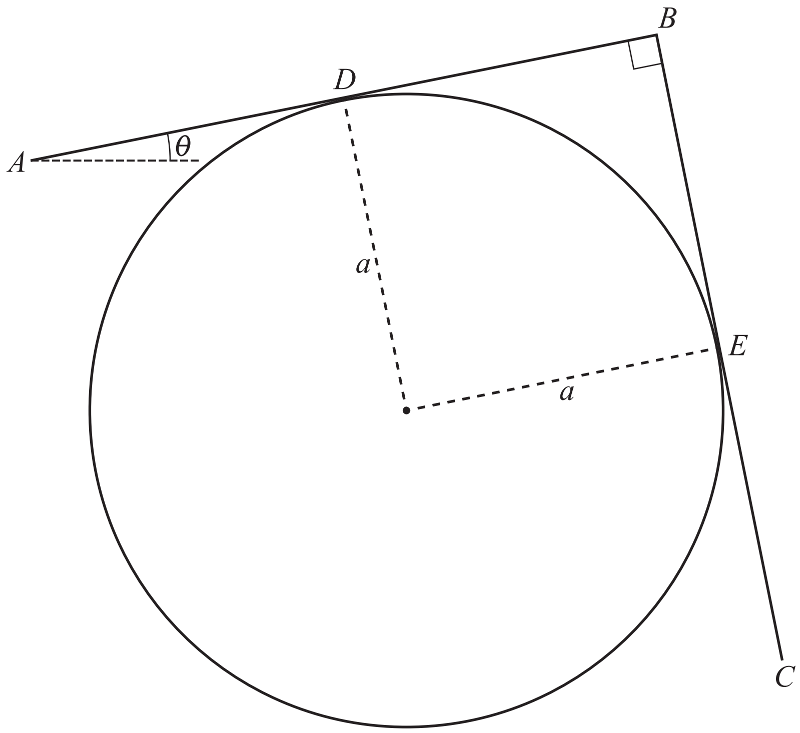
(b) Find the size of angle β. [4]

7



A uniform cylinder with a rough surface and of radius a is fixed with its axis horizontal. Two identical uniform rods AB and BC, each of weight W and length $2a$, are rigidly joined at B with AB perpendicular to BC. The rods rest on the cylinder in a vertical plane perpendicular to the axis of the cylinder with AB at an angle θ to the horizontal. D and E are the midpoints of AB and BC respectively and also the points of contact of the rods with the cylinder (see diagram). The rods are about to slip in a clockwise direction. The coefficient of friction between each rod and the cylinder is μ.

The normal reaction between AB and the cylinder is R and the normal reaction between BC and the cylinder is N.

(a) Find the ratio $R : N$ in terms of μ. [6]

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(b) Given that $\mu = \frac{1}{3}$, find the value of $\tan\theta$. [3]

Additional page

If you use the following page to complete the answer to any question, the question number must be clearly shown.

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Cambridge Assessment International Education

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FURTHER MATHEMATICS **9231/33**

Paper 3 Further Mechanics **May/June 2022**

MARK SCHEME

Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2022 series for most Cambridge IGCSE, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:
Marks must be awarded in line with:
- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:
Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:
Marks must be awarded **positively**:
- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:
Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

<table>
<tr><th>Question</th><th>Answer</th><th>Marks</th><th>Guidance</th></tr>
<tr><td rowspan="5">1</td><td>
<table>
<tr><th></th><th>Area</th><th>Distance from Oy</th></tr>
<tr><td>Triangle OCD</td><td>6</td><td>2</td></tr>
<tr><td>Rectangle DEBC</td><td>24</td><td>6</td></tr>
<tr><td>Triangle BAE</td><td>12</td><td>11</td></tr>
<tr><td>Trapezium OCBA</td><td>42</td><td>$\bar{x}$</td></tr>
</table>
where D is point with coordinates (3, 0) and E is a point with coordinates (9, 0).</td><td>M1</td><td>Attempt at moments equation with all necessary terms.

Other options possible for RHS of moments equation, for example:
(1) OAC: 30×6 and ABC: 12×9
(2) OBC: 12×4 and OAB: 30×8
(3) Subtraction: $60\times 7.5-6\times 1-12\times 13$</td></tr>
<tr><td>Parts that would give correct total area 42</td><td>B1</td><td></td></tr>
<tr><td>Moments about Oy
$42\bar{x}=6\times 2+24\times 6+12\times 11\quad (=288)$</td><td>A1</td><td>Correct equation.</td></tr>
<tr><td>$\bar{x}=\dfrac{288}{42}=6.86$</td><td>A1</td><td></td></tr>
<tr><td></td><td>4</td><td></td></tr>
</table>

Question	Answer	Marks	Guidance
2	$\frac{1}{2}.\frac{\frac{4}{3}mg}{a}x^2$	B1	EPE term correct
	$\frac{1}{3}mgx$	B1	Work term correct
	Loss in KE = gain in EPE + work done against friction $\frac{1}{2}mv^2 = \frac{1}{2}\times\frac{\frac{4}{3}mg}{a}x^2 + \frac{1}{3}mgx$	M1	Energy equation with 3 terms, allow sign error.
	$\frac{1}{2}\times\frac{1}{4}ga = \frac{\frac{2}{3}g}{a}x^2 + \frac{1}{3}gx$ $16x^2 + 8ax - 3a^2 = 0$ $(4x - a)(4x + 3a) = 0$	M1	Obtain and attempt to solve a 3-term quadratic equation.
	$x = \frac{1}{4}a$	A1	
		5	

Question	Answer	Marks	Guidance
3(a)	Components of velocity : $\rightarrow 25\cos\theta$ $\uparrow 25\sin\theta - 2g$	**B1**	
	Speed = $\sqrt{(25\cos\theta)^2 + (25\sin\theta - 2g)^2}$	**M1 A1**	Expression for speed or square of speed.
	$(25\cos\theta)^2 + (25\sin\theta - 2g)^2 = 15^2$ $625 - 100g\sin\theta + 4g^2 = 225$	**M1**	Attempt to solve and find value for $\sin\theta$
	$\sin\theta = \frac{800}{1000} = \frac{4}{5}$	**A1**	
		5	
3(b)	Time of flight $= \left(\frac{2\times 25\sin\theta}{g}\right) = 4$ (s)	**B1**	
	Range = $\frac{2\times 25\sin\theta}{g} \times 25\cos\theta$	**M1**	Any equivalent method.
	Range = 60 (m)	**A1**	CWO
	Alternative method for question 3(b)		
	$y = \frac{4}{3}x - \frac{1}{45}x^2$	**B1**	Equation of trajectory..
	Substitute $y = 0$ and solve	**M1**	
	60 (m)	**A1**	
		3	

Question	Answer	Marks	Guidance
4(a)	$\frac{1}{2}mv^2 - \frac{1}{2}mu^2 = mga(\cos\theta + \cos\alpha)$	**M1**	Energy equation with all necessary terms, GPE terms must be resolved, allow sin/cos mix, allow sign error.
	$\frac{1}{2}m(2u)^2 - \frac{1}{2}mu^2 = mga(\cos\theta + \cos\alpha)$	**A1**	$2u$ may be substituted later. Implied by $\frac{3}{2}\times\frac{2}{3}ag = ga(\cos\theta + \cos\alpha)$
	At A, $T + mg\cos\theta = \frac{m}{a}u^2$	**B1**	N2L
	Also, $10T - mg\cos\alpha = \frac{m}{a}4u^2$	**B1**	N2L and use of tension ($10T$).
	Use all three (two N2L and energy) equations to find T in terms of m and g only.	**M1**	Might see $9T - mg(\cos\theta + \cos\alpha) = \frac{3m}{a}\times\frac{2}{3}ga$ $(\cos\theta + \cos\alpha) = 1$ $(10\cos\theta + \cos\alpha) = 4$
	$T = \frac{1}{3}mg$	**A1**	
		6	
4(b)	Substitute back, $10\times\frac{1}{3}mg - mg\cos\alpha = \frac{4m}{a}\times\frac{2}{3}ga$	**M1**	Any appropriate method to obtain $\cos\alpha$.
	$\cos\alpha = \frac{2}{3}$	**A1**	
		2	

Question	Answer	Marks	Guidance
5(a)	$4v\frac{dv}{dx} = -\left(4e^{-x} + 12\right)e^{-x}$	B1	
	$\frac{1}{2}v^2 = \frac{1}{2}e^{-2x} + 3e^{-x}\left(+A\right)$	M1	Expression of the correct form.
	$v = 4, \quad x = 0, \quad A = \frac{9}{2}$	A1	
	$v^2 = e^{-2x} + 6e^{-x} + 9 = \left(3 + e^{-x}\right)^2$ $v = 3 + e^{-x} = \frac{1 + 3e^x}{e^x}$	A1	AG Must see the factorisation. Condone lack of justification for taking positive square root.
		4	
5(b)	$\frac{dx}{dt} = \frac{1 + 3e^x}{e^x}$ so $\int \frac{e^x}{3e^x + 1}dx = \int 1\ dt$ $\frac{1}{3}\ln\left(3e^x + 1\right) = t\left(+B\right)$	M1* A1	Integration to obtain ln term Correct answer with constant of integration
	$t = 0, \quad x = 0, \quad B = \frac{1}{3}\ln 4$ $3t = \ln\frac{3e^x + 1}{4}$	DM1	Find the constant and substitute into their general solution.
	$x = \ln\left(\frac{4}{3}e^{3t} - \frac{1}{3}\right)$	A1	OE
		4	

Question	Answer	Marks	Guidance
6(a)	Let v be speed of rebound from 1st collision: Energy loss: $\frac{1}{2}mv^2 = \frac{2}{5} \times \frac{1}{2}mu^2, \quad v^2 = \frac{2}{5}u^2$	B1	Energy loss.
	$v\cos\alpha = u\cos\theta$ $v\sin\alpha = eu\sin\theta$	B1	Both.
	Combine to form equation in e only $\frac{2}{5} = \frac{1}{5} + e^2 \times \frac{4}{5}$	M1	$v^2 = (u\cos\theta)^2 + (eu\sin\theta)^2$
	$e = \frac{1}{2}$	A1	
		4	
6(b)	$\tan\alpha = e\tan\theta$, so $\tan\alpha = 1, \quad \alpha = 45°$	B1	
	For 2nd collision $w\cos\beta = v\cos(180 - 60 - \alpha)$ $w\sin\beta = ev\sin(180 - 60 - \alpha)$	M1	Both. May be implied by the A1.
	$\tan\beta = e\tan(120 - \textit{their}\ \alpha)$	M1	Divide to find β.
	$\beta = 61.8°$	A1	
		4	

Question	Answer	Marks	Guidance
7(a)	Frictional force = $\mu\times$ normal reaction at D and E	B1	$F_{AB}=\mu R,\ F_{BC}=\mu N$
	Moments about B, $Na-Ra=Wa(\sin\theta-\cos\theta)$ Moments about centre, $F_{AB}a+F_{BC}a=Wa(\cos\theta-\sin\theta)$ Moments about D, $F_{BC}a+Na=Wa(\cos\theta+\sin\theta)$ Moments about E, $Ra-F_{AB}a=Wa(\cos\theta+\sin\theta)$	B1	One moments equation about any point involving all relevant forces, resolved if necessary (AEF).
	Parallel to AB, $N-F_{AB}=W\sin\theta+W\sin\theta$ Perpendicular to AB, $F_{BC}+R=W\cos\theta+W\cos\theta$	B1	Two resolutions: all relevant terms, different frictional forces [Vertical: $R\cos\theta+F_{BC}\cos\theta+N\sin\theta=F_{AB}\sin\theta+W+W$ Horizontal: $F_{BC}\sin\theta+F_{AB}\cos\theta+R\sin\theta=N\cos\theta$] Alternative approach using two moments equations can earn the B1B1
	$N-R=\frac{1}{2}\left((1-\mu)N-(1+\mu)R\right)$	M1	Combine appropriate equations.
	$N\left(1-\frac{1}{2}(1-\mu)\right)=R\left(1-\frac{1}{2}(1+\mu)\right)$ $N\left(\frac{1}{2}+\frac{1}{2}\mu\right)=R\left(\frac{1}{2}-\frac{1}{2}\mu\right)$	M1	Collect terms to obtain ratio/fraction in terms of μ only (CWO), any equivalent simplified form.
	$R:N=1+\mu:1-\mu$	A1	
		6	

Question	Answer	Marks	Guidance
7(b)	Divide resolution equations: $\tan\theta = \dfrac{N - \mu R}{\mu N + R}$	M1	Must include μ terms.
	Use $R = 2N$: $\tan\theta = \dfrac{\frac{1}{3}N}{\frac{7}{3}N}$	M1	FT their answer to part **(a)**.
	$\tan\theta = \dfrac{1}{7}$	A1	
		3	

Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

3224279643

FURTHER MATHEMATICS **9231/31**

Paper 3 Further Mechanics **October/November 2021**

1 hour 30 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.
- Where a numerical value for the acceleration due to gravity (g) is needed, use $10\,\mathrm{m\,s^{-2}}$.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Any blank pages are indicated.

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1 One end of a light elastic string, of natural length a and modulus of elasticity $3mg$, is attached to a fixed point O on a smooth horizontal plane. A particle P of mass m is attached to the other end of the string and moves in a horizontal circle with centre O. The speed of P is $\sqrt{\frac{4}{3}ga}$.

Find the extension of the string. [4]

2 A particle P of mass m kg moves along a horizontal straight line with acceleration $a\,\mathrm{m\,s^{-2}}$ given by

$$a = \frac{v(1-2t^2)}{t},$$

where $v\,\mathrm{m\,s^{-1}}$ is the velocity of P at time t s.

(a) Find an expression for v in terms of t and an arbitrary constant. [3]

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(b) Given that $a = 5$ when $t = 1$, find an expression, in terms of m and t, for the horizontal force acting on P at time t. [3]

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3 A light elastic string has natural length a and modulus of elasticity $12mg$. One end of the string is attached to a fixed point O. The other end of the string is attached to a particle of mass m. The particle hangs in equilibrium vertically below O. The particle is pulled vertically down and released from rest with the extension of the string equal to e, where $e > \frac{1}{3}a$. In the subsequent motion the particle has speed $\sqrt{2ga}$ when it has ascended a distance $\frac{1}{3}a$.

Find e in terms of a. [6]

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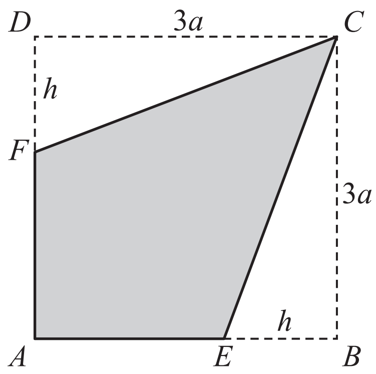

A uniform lamina $AECF$ is formed by removing two identical triangles BCE and CDF from a square lamina $ABCD$. The square has side $3a$ and $EB = DF = h$ (see diagram).

(a) Find the distance of the centre of mass of the lamina $AECF$ from AD and from AB, giving your answers in terms of a and h. [5]

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The lamina $AECF$ is placed vertically on its edge AE on a horizontal plane.

(b) Find, in terms of a, the set of values of h for which the lamina remains in equilibrium. [3]

5 A particle P is projected from a point O on a horizontal plane and moves freely under gravity. Its initial speed is $u\,\mathrm{m\,s^{-1}}$ and its angle of projection is $\sin^{-1}(\frac{4}{5})$ above the horizontal. At time 8 s after projection, P is at the point A. At time 32 s after projection, P is at the point B. The direction of motion of P at B is perpendicular to its direction of motion at A.

Find the value of u. [7]

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6 A particle P, of mass m, is attached to one end of a light inextensible string of length a. The other end of the string is attached to a fixed point O. The particle P moves in complete vertical circles about O with the string taut. The points A and B are on the path of P with AB a diameter of the circle. OA makes an angle θ with the downward vertical through O and OB makes an angle θ with the upward vertical through O. The speed of P when it is at A is $\sqrt{5ag}$.

The ratio of the tension in the string when P is at A to the tension in the string when P is at B is 9 : 5.

(a) Find the value of $\cos\theta$. [6]

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(b) Find, in terms of a and g, the greatest speed of P during its motion. [2]

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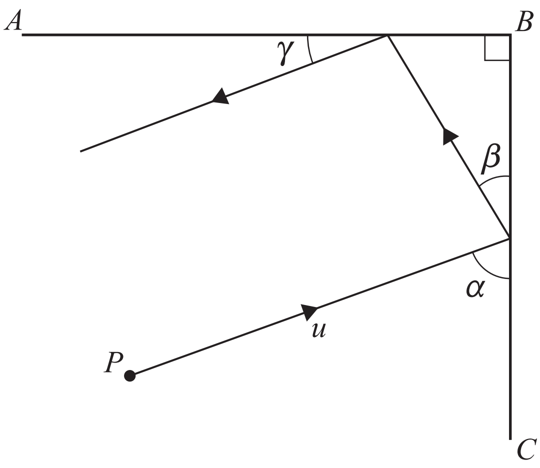

The smooth vertical walls AB and CB are at right angles to each other. A particle P is moving with speed u on a smooth horizontal floor and strikes the wall CB at an angle α. It rebounds at an angle β to the wall CB. The particle then strikes the wall AB and rebounds at an angle γ to that wall (see diagram). The coefficient of restitution between each wall and P is e.

(a) Show that $\tan\beta = e\tan\alpha$. [3]

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(b) Express γ in terms of α and explain what this result means about the final direction of motion of P. [4]

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As a result of the two impacts the particle loses $\frac{8}{9}$ of its initial kinetic energy.

(c) Given that $\alpha + \beta = 90°$, find the value of e and the value of $\tan\alpha$. [4]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

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Cambridge Assessment International Education

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FURTHER MATHEMATICS **9231/31**

Paper 3 Further Mechanics **October/November 2021**

MARK SCHEME

Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

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WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	$T = \dfrac{3mgx}{a}$	**B1**	Their tensions equated to obtain a quadratic equation, CAO.
	$T = \dfrac{4mga}{3(a+x)}$	**B1**	
	$9x^2 + 9ax - 4a^2 = 0$ leading to $(3x-a)(3x+4a) = 0$	**M1**	
	$x = \dfrac{1}{3}a$	**A1**	
		4	
2(a)	Separate variables and integrate: $\dfrac{dv}{v} = \left(\dfrac{1-2t^2}{t}\right)dt$ so $\ln\|v\| = \ln t - t^2 + c$	**M1 A1**	Must include logs. Condone missing modulus.
	$\|v\| = Ate^{-t^2}$, $-v = Ate^{-t^2}$, $v = -Ate^{-t^2}$	**A1**	CAO.
		3	

Question	Answer	Marks	Guidance
2(b)	$a=\dfrac{-Ate^{-t^2}\left(1-2t^2\right)}{t}=-Ae^{-t^2}\left(1-2t^2\right)$	**M1**	Substituting their answer to part **(a)** into given formula
	$t=1,\ a=5\quad\left(A=5e\right)$	**M1**	Use initial condition.
	Force $=5me^{1-t^2}\left(2t^2-1\right)$	**A1**	Use N2L, correct work only.
	Alternative method for question 2(b)		
	$a=\dfrac{v\left(1-2t^2\right)}{t}$ substitute $t=1,\ a=5$ so $v=-5$	**M1**	Use initial condition. Use N2L, correct work only.
	Substituting in their answer to part **(a)** so $\left(A=5e\right)$	**M1**	
	Force $=5me^{1-t^2}\left(2t^2-1\right)$	**A1**	
		3	

Question	Answer	Marks	Guidance
3	Loss in EPE = $\frac{1}{2}\times\frac{12mge^2}{a}-\frac{1}{2}\times\frac{12mg}{a}\times\left(e-\frac{a}{3}\right)^2$ $\left(=\frac{2mg}{3}(6e-a)\right)$	**B1**	Either term correct.
	Gain in KE = $\frac{1}{2}mv^2$ and Gain in GPE = $\frac{mga}{3}$	**B1**	
	Gain in KE + Gain in GPE = Loss in EPE	**M1**	KE, GPE and at least one EPE term.
	$\frac{1}{2}mv^2+\frac{mga}{3}=\frac{2mg}{3}(6e-a)$	**A1**	All terms correct.
	Simplify to a linear equation in *e*.	**M1**	
	$e=\frac{1}{2}a$	**A1**	
		6	

Question	Answer	Marks	Guidance
4(a)	<table><tr><td></td><td>Area</td><td>Centre of mass from *AD*</td></tr><tr><td>Square</td><td>$9a^2$</td><td>$\frac{3}{2}a$</td></tr><tr><td>*CDF*</td><td>$\frac{3}{2}ah$</td><td>a</td></tr><tr><td>*BEC*</td><td>$\frac{3}{2}ah$</td><td>$3a-\frac{1}{3}h$</td></tr><tr><td>Resulting *AEFC*</td><td>$9a^2-3ah$</td><td>$\bar{x}$</td></tr></table>	**M1**	Attempt at moments with three terms.
	Taking moments about *AD*: $\left(9a^2-3ah\right)\ \bar{x}=\left(9a^2\times\frac{3}{2}a\right)-\left(\frac{3}{2}ah\times a\right)-\left(\frac{3}{2}ah\times\left(3a-\frac{1}{3}h\right)\right)$	**A1** **A1**	Two terms correct. All correct.
	$\bar{x}=\frac{27a^2-12ah+h^2}{6(3a-h)}\ \left(=\frac{9a-h}{6}\right)$	**A1**	AEF
	$\bar{y}=\bar{x}$	**B1**	By symmetry or equal to their $\bar{x}$.
		5	
4(b)	For equilibrium, $\bar{x}\leqslant 3a-h$ $27a^2-12ah+h^2\leqslant 6(3a-h)^2$	**B1**	Accept strict inequality.
	$27a^2-24ah+5h^2\geqslant 0$	**M1**	Homogeneous 3-term quadratic inequality.
	$h\leqslant\frac{9}{5}a$	**A1**	CAO.
		3	

Question	Answer	Marks	Guidance
5	At A: $\uparrow u\sin\theta - 8g \quad \rightarrow u\cos\theta$	**M1**	Both.
	$\tan\alpha = \dfrac{u\sin\theta - 8g}{u\cos\theta}$	**A1**	
	At B: $\uparrow u\sin\theta - 32g \quad \rightarrow u\cos\theta$	**M1**	Both.
	$\tan\beta = \dfrac{u\sin\theta - 32g}{u\cos\theta}$	**A1**	
	$\dfrac{u\sin\theta - 8g}{u\cos\theta} \times \dfrac{u\sin\theta - 32g}{u\cos\theta} = -1$	**B1**	Perpendicular directions, so $\tan\alpha \times \tan\beta = -1$.
	$u^2 - 320u + 25600 = 0$	**M1**	Simplify to a quadratic in u.
	$u = 160$	**A1**	
		7	

Question	Answer	Marks	Guidance
6(a)	At A $T_A - mg\cos\theta = m\times\frac{5ag}{a}$	B1	N2L
	At B $T_B + mg\cos\theta = m\times\frac{v^2}{a}$	B1	N2L
	$\frac{1}{2}m\times 5ag - \frac{1}{2}mv^2 = mga\times 2\cos\theta$	M1	Energy equation with correct number of terms.
	$v^2 = 5ag - 4ga\cos\theta$	A1	Accept multiplied by m and/or divided by a.
	Use ratio of tensions = 9 : 5	M1	Use ratio and simplify to an expression in $\cos\theta$.
	$\cos\theta = \frac{2}{5}$	A1	CAO
		6	
6(b)	Greatest speed at lowest point $-\frac{1}{2}m\times 5ag + \frac{1}{2}mV^2 = mga\times(1-\cos\theta)$	M1	Energy equation including lowest point, correct number of terms.
	$V = \sqrt{\frac{31ag}{5}}$	A1 FT	Ft their $\cos\theta$ from part **(a)**.
		2	

Question	Answer	Marks	Guidance
7(a)	$u\cos\alpha = v\cos\beta$	**M1**	
	$eu\sin\alpha = v\sin\beta$	**M1**	
	Divide: $\tan\beta = e\tan\alpha$	**A1**	AG. Must see divide OE.
		3	
7(b)	$v\sin\beta = w\cos\gamma \quad (= eu\sin\alpha)$	**M1**	
	$ev\cos\beta = w\sin\gamma \quad (= eu\cos\alpha)$	**M1**	
	Divide: $\tan\gamma = 1/\tan\alpha$: $\gamma = 90° - \alpha$	***A1**	
	After second rebound, direction of motion is parallel to initial path.	**DB1**	
		4	
7(c)	Final KE = $\frac{1}{2}m\left((eu\sin\alpha)^2 + (eu\cos\alpha)^2\right) \quad \left(= \frac{1}{2}me^2u^2\right)$	**M1**	Energy expression in terms of u.
	So $\frac{1}{2}me^2u^2 = \frac{1}{9}\times\frac{1}{2}mu^2$ giving $e = \frac{1}{3}$	**A1**	
	Part **(a)** gives $\tan(90-\alpha) = e\tan\alpha$	**M1**	
	So $\tan\alpha = \sqrt{3}$	**A1**	
		4	

Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME			
CENTRE NUMBER		CANDIDATE NUMBER	

7215176628

FURTHER MATHEMATICS **9231/32**

Paper 3 Further Mechanics **October/November 2021**

1 hour 30 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS
- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.
- Where a numerical value for the acceleration due to gravity (g) is needed, use $10\,\mathrm{m\,s^{-2}}$.

INFORMATION
- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Any blank pages are indicated.

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1 A particle is projected with speed u at an angle α above the horizontal from a point O on a horizontal plane. The particle moves freely under gravity.

(a) Write down the horizontal and vertical components of the velocity of the particle at time T after projection. [2]

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At time T after projection, the direction of motion of the particle is perpendicular to the direction of projection.

(b) Express T in terms of u, g and α. [2]

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(c) Deduce that $T > \frac{u}{g}$. [1]

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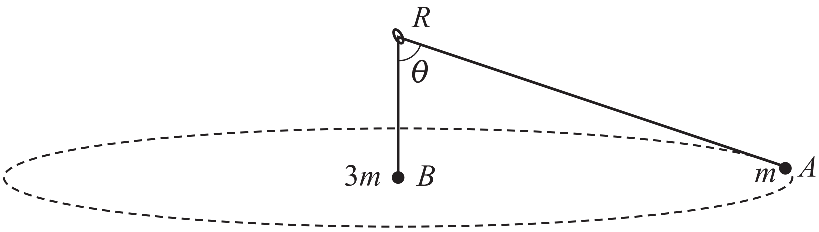
2 A light spring AB has natural length a and modulus of elasticity $5mg$. The end A of the spring is attached to a fixed point on a smooth horizontal surface. A particle P of mass m is attached to the end B of the spring. The spring and particle P are at rest on the surface.

Another particle Q of mass km is moving with speed $\sqrt{4ga}$ along the horizontal surface towards P in the direction BA. The particles P and Q collide directly and coalesce. In the subsequent motion the greatest amount by which the spring is compressed is $\frac{1}{5}a$.

Find the value of k. [6]

3



Particles A and B, of masses m and $3m$ respectively, are connected by a light inextensible string of length a that passes through a fixed smooth ring R. Particle B hangs in equilibrium vertically below the ring. Particle A moves in horizontal circles with speed v. Particles A and B are at the same horizontal level. The angle between AR and BR is θ (see diagram).

(a) Show that $\cos\theta = \frac{1}{3}$. [2]

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(b) Find an expression for v in terms of a and g. [4]

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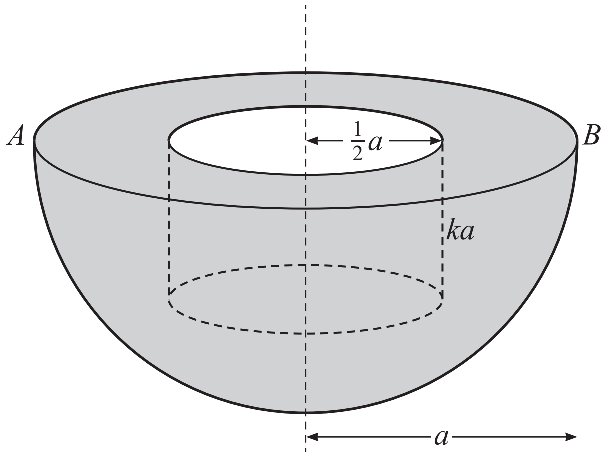

An object is formed by removing a solid cylinder, of height ka and radius $\frac{1}{2}a$, from a uniform solid hemisphere of radius a. The axes of symmetry of the hemisphere and the cylinder coincide and one circular face of the cylinder coincides with the plane face of the hemisphere. AB is a diameter of the circular face of the hemisphere (see diagram).

(a) Show that the distance of the centre of mass of the object from AB is $\dfrac{3a(2-k^2)}{2(8-3k)}$. [4]

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When the object is freely suspended from the point A, the line AB makes an angle θ with the downward vertical, where $\tan\theta = \frac{7}{18}$.

(b) Find the possible values of k. [3]

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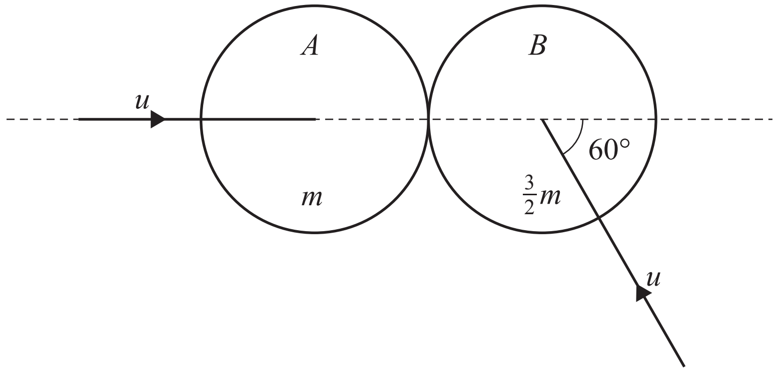

Two uniform smooth spheres A and B of equal radii have masses m and $\frac{3}{2}m$ respectively. The two spheres are each moving with speed u on a horizontal surface when they collide. Immediately before the collision A's direction of motion is along the line of centres, and B's direction of motion makes an angle of 60° with the line of centres (see diagram). The coefficient of restitution between the spheres is $\frac{2}{3}$.

(a) Find the angle through which the direction of motion of B is deflected by the collision. [6]

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(b) Find the loss in the total kinetic energy of the system as a result of the collision. [3]

6 A particle P of mass 2 kg moves along a horizontal straight line. The point O is a fixed point on this line. At time t s the velocity of P is $v\,\text{m s}^{-1}$ and the displacement of P from O is x m.

A force of magnitude $\left(8x - \frac{128}{x^3}\right)$ N acts on P in the direction OP. When $t = 0$, $x = 8$ and $v = -15$.

(a) Show that $v = -\frac{2}{x}(x^2 - 4)$. [5]

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(b) Find an expression for x in terms of t. [4]

7 One end of a light inextensible string of length a is attached to a fixed point O. The other end of the string is attached to a particle P of mass m. The particle P is held vertically below O with the string taut and then projected horizontally. When the string makes an angle of 60° with the upward vertical, P becomes detached from the string. In its subsequent motion, P passes through the point A which is a distance a vertically above O.

(a) The speed of P when it becomes detached from the string is V. Use the equation of the trajectory of a projectile to find V in terms of a and g. [4]

(b) Find, in terms of m and g, the tension in the string immediately after P is initially projected horizontally. [4]

Additional Page

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Paper 3 Further Mechanics **October/November 2021**

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Question	Answer	Marks	Guidance
1(a)	Velocity: $\rightarrow u\cos\alpha$	B1	
	$\uparrow u\sin\alpha - gT$	B1	Allow 10 for g. Must be T.
		2	
1(b)	$\dfrac{u\cos\alpha}{u\sin\alpha - gT} = -\dfrac{\sin\alpha}{\cos\alpha}$ oe	M1 FT	Allow missing minus sign on RHS for M1. FT from **(a)**.
	$T = \dfrac{u}{g\sin\alpha}$	A1	
		2	
1(c)	$\sin\alpha < 1$ giving $T > \dfrac{u}{g}$	B1	AG
		1	

Question	Answer	Marks	Guidance
2	At the collision of P and Q: $(m+km)v = kmu$	**M1**	Momentum conserved, allow missing k on RHS.
	So $v = \dfrac{k\sqrt{4ga}}{(1+k)}$	**A1**	
	EPE $= \dfrac{1}{2} \times \dfrac{5mg}{a} \times \left(\dfrac{a}{5}\right)^2 \quad \left(= \dfrac{mga}{10}\right)$	**B1**	
	Loss in KE = Gain in EPE: $\dfrac{1}{2}m(k+1)v^2 = \dfrac{1}{2} \times \dfrac{5mg}{a} \times \left(\dfrac{a}{5}\right)^2$	**M1**	Energy equation, LHS correct, EPE dimensionally correct.
	Substitute for v and rearrange to form quadratic equation in k $20k^2 = 1 + k$	**M1**	
	$k = \dfrac{1}{4}$	**A1**	
		6	

Question	Answer	Marks	Guidance
3(a)	$T = 3mg$ and $T\cos\theta = mg$	M1	Must see both of these separately.
	Combining, $\cos\theta = \frac{1}{3}$	A1	At least one step of working, AG.
		2	
3(b)	$(\cos\theta = \frac{a-x}{x}$, where $x = AR)$ $AR = \frac{3}{4}a$ or $BR = \frac{1}{4}a$ or radius = $\frac{a}{\sqrt{2}}$	B1	$\left(\sin\theta = \frac{\sqrt{8}}{3}\right)$
	$T\sin\theta = \frac{mv^2}{r}$	M1	
	Combining to find an equation in v^2, a and g only.	DM1	
	$v^2 = 2ga$, $v = \sqrt{2ga}$	A1	
		4	

Question	Answer	Marks	Guidance
4(a)	<table><tr><td></td><td>Volume</td><td>Centre of mass from *AB*</td></tr><tr><td>Hemisphere</td><td>$\frac{2}{3}\pi a^3$</td><td>$\frac{3}{8}a$</td></tr><tr><td>Cylinder</td><td>$\pi ka(\frac{a}{2})^2$</td><td>$\frac{ka}{2}$</td></tr><tr><td>Remainder</td><td>$\frac{2}{3}\pi a^3 - \pi ka\left(\frac{a}{2}\right)^2$</td><td>$\bar{x}$</td></tr></table>	**M1**	Attempt at moments, 3 terms.
	Taking moments about *AB*: $\left(\frac{2}{3}\pi a^3 - \pi ka\left(\frac{a}{2}\right)^2\right)\times\bar{x} = \left(\frac{2}{3}\pi a^3 \times \frac{3}{8}a\right) - \left(\pi ka\left(\frac{a}{2}\right)^2 \times \frac{ka}{2}\right)$	**A1** **A1**	Any 2 terms correct. All correct.
	$\bar{x} = \frac{3a\left(2-k^2\right)}{2(8-3k)}$	**A1**	Shown convincingly, AG.
		4	
4(b)	$\tan\theta = \frac{\bar{x}}{a}$ $\frac{3\left(2-k^2\right)}{2(8-3k)} = \frac{7}{18}$	**B1**	
	$27k^2 - 21k + 2 = 0$	**M1**	Rearrange to form quadratic.
	$k = \frac{2}{3}$ and $k = \frac{1}{9}$	**A1**	Both answers correct.
		3	

Question	Answer	Marks	Guidance
5(a)	Let speeds of *A* and *B* along line of centres after collision be v_1 and v_2 $mv_1 + \frac{3}{2}mv_2 = -\frac{3}{2}mu\cos 60° + mu \quad \left(=\frac{u}{4}\right)$	**M1**	Momentum with masses correct.
	$v_2 - v_1 = -\frac{2}{3}(-u\cos 60° - u) \quad (=u)$	**M1**	Restitution, with consistent signs on LHS.
	$\left(v_1 = -\frac{1}{2}u\right) \quad v_2 = \frac{1}{2}u$	**A1**	
	Perpendicular to line of centres, speed of B is $u\sin 60° = \frac{\sqrt{3}}{2}u$	**B1**	
	Direction of B is now 60° above line of centres.	**M1**	
	Angle of deflection is 60°.	**A1 FT**	FT (120° – *their* direction of B angle)
		6	
5(b)	KE before = $\frac{1}{2}mu^2 + \frac{1}{2}.\frac{3m}{2}u^2 = \frac{5}{4}mu^2$	**B1**	
	KE after = $\frac{1}{2}m\left(\frac{u}{2}\right)^2 + \frac{1}{2}.\frac{3m}{2}\left(\left(\frac{u}{2}\right)^2 + \left(\frac{\sqrt{3}u}{2}\right)^2\right) \quad \left(=\frac{7}{8}mu^2\right)$	**B1 FT**	FT only *their* speeds from **(a)**
	Loss in KE = $\frac{3}{8}mu^2$	**B1**	
		3	

Question	Answer	Marks	Guidance
6(a)	$2v\frac{dv}{dx} = 8x - \frac{128}{x^3}$	M1	Separate variables and integrate, + c not required for M1.
	$v^2 = 4x^2 + 64x^{-2} + c$	A1	OE.
	$x = 8$, $v = -15$ and $c = -32$	M1	Use initial condition.
	$v^2 = \frac{4}{x^2}\left(x^4 - 8x^2 + 16\right)$ or $4x^2 - 32 + \frac{64}{x^2}$	A1	Correct expression for v^2, AEF.
	$v^2 = \frac{4}{x^2}\left(x^2 - 4\right)^2$ giving $v = -\frac{2}{x}\left(x^2 - 4\right)$	A1	Convincingly shown, e.g. v is negative initially, AG.
		5	
6(b)	$\frac{1}{2}\ln(x^2 - 4) = -2t\left(+A\right)$	M1	Use $v = \frac{dx}{dt}$ and integrate.
	$t = 0,\ x = 8,\ \ A = \frac{1}{2}\ln 60$	DM1	Use initial condition.
	$\frac{1}{2}\ln\left(\frac{x^2 - 4}{60}\right) = -2t$ giving $\frac{x^2 - 4}{60} = e^{-4t}$	M1	Remove log.
	$x = \sqrt{4 + 60e^{-4t}}$	A1	CAO
		4	

Question	Answer	Marks	Guidance
7(a)	Coordinates of A: $x = a\sin 60$, $y = a - a\cos 60$	B1	
	$\frac{a}{2} = \frac{a\sqrt{3}}{2}\sqrt{3} - \frac{g\frac{\left(a\sqrt{3}\right)^2}{2^2}}{2V^2.\frac{1}{4}}$	M1	Substitute *their* (x, y) into correct trajectory equation.
	Rearrange to find V^2.	M1	
	$V^2 = \frac{3}{2}ag$, $V = \sqrt{\frac{3}{2}ag}$	A1	
		4	
7(b)	$\frac{1}{2}mu^2 - \frac{1}{2}mV^2 = mga(1 + \cos 60)$	M1	Energy equation.
	$u^2 = \frac{9}{2}ag$	A1	u is the speed at P.
	$T - mg = \frac{m}{a}u^2$	M1	N2L
	$T = \frac{11}{2}mg$	A1	
		4	

Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

7978932017

FURTHER MATHEMATICS **9231/32**

Paper 3 Further Mechanics **May/June 2021**

1 hour 30 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS
- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.
- Where a numerical value for the acceleration due to gravity (g) is needed, use $10\,\mathrm{m\,s^{-2}}$.

INFORMATION
- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Any blank pages are indicated.

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1 A particle P of mass 1 kg is moving along a straight line against a resistive force of magnitude $\frac{10\sqrt{v}}{(t+1)^2}$ N, where $v\,\mathrm{m\,s^{-1}}$ is the speed of P at time t s. When $t = 0$, $v = 25$.

Find an expression for v in terms of t. [5]

2 A hollow hemispherical bowl of radius a has a smooth inner surface and is fixed with its axis vertical. A particle P of mass m moves in horizontal circles on the inner surface of the bowl, at a height x above the lowest point of the bowl. The speed of P is $\sqrt{\frac{8}{3}ga}$.

Find x in terms of a. [6]

3 One end of a light elastic string, of natural length a and modulus of elasticity kmg, is attached to a fixed point A. The other end of the string is attached to a particle P of mass $4m$. The particle P hangs in equilibrium a distance x vertically below A.

(a) Show that $k = \frac{4a}{x-a}$. [1]

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An additional particle, of mass $2m$, is now attached to P and the combined particle is released from rest at the original equilibrium position of P. When the combined particle has descended a distance $\frac{1}{3}a$, its speed is $\frac{1}{3}\sqrt{ga}$.

(b) Find x in terms of a. [6]

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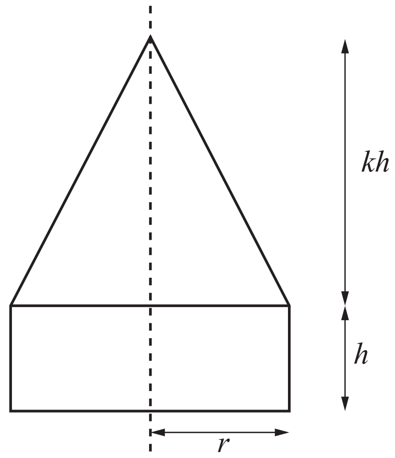


A uniform solid circular cone has vertical height kh and radius r. A uniform solid cylinder has height h and radius r. The base of the cone is joined to one of the circular faces of the cylinder so that the axes of symmetry of the two solids coincide (see diagram, which shows a cross-section). The cone and the cylinder are made of the same material.

(a) Show that the distance of the centre of mass of the combined solid from the base of the cylinder is $\dfrac{h(k^2+4k+6)}{4(3+k)}$. [4]

The solid is placed on a plane that is inclined to the horizontal at an angle θ. The base of the cylinder is in contact with the plane. The plane is sufficiently rough to prevent sliding. It is given that $3h = 2r$ and that the solid is on the point of toppling when $\tan\theta = \frac{4}{3}$.

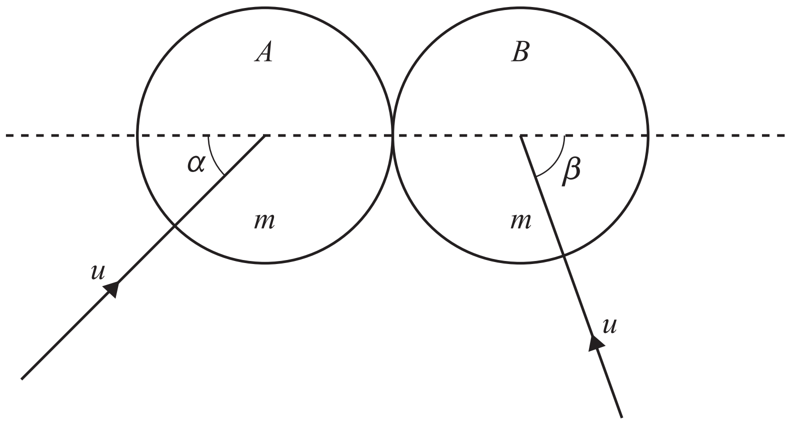
(b) Find the value of k. [3]

5 A particle P of mass m is attached to one end of a light inextensible string of length a. The other end of the string is attached to a fixed point O. The particle completes vertical circles with centre O. The points A and B are on the path of P, both on the same side of the vertical through O. OA makes an angle θ with the downward vertical through O and OB makes an angle θ with the upward vertical through O.

The speed of P when it is at A is u and the speed of P when it is at B is $\sqrt{ag}$. The tensions in the string at A and B are T_A and T_B respectively. It is given that $T_A = 7T_B$.

Find the value of θ and find an expression for u in terms of a and g. [8]

6



Two uniform smooth spheres A and B of equal radii each have mass m. The two spheres are each moving with speed u on a horizontal surface when they collide. Immediately before the collision, A's direction of motion makes an angle α with the line of centres, and B's direction of motion makes an angle β with the line of centres (see diagram). The coefficient of restitution between the spheres is $\frac{1}{3}$ and $2\cos\beta = \cos\alpha$.

(a) Show that the direction of motion of A after the collision is perpendicular to the line of centres. [4]

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The total kinetic energy of the spheres after the collision is $\frac{3}{4}mu^2$.

(b) Find the value of α. [4]

7 A particle P is projected from a point O on a horizontal plane and moves freely under gravity. The initial velocity of P is $100\,\mathrm{m\,s^{-1}}$ at an angle θ above the horizontal, where $\tan\theta = \frac{4}{3}$. The two times at which P's height above the plane is H m differ by 10 s.

(a) Find the value of H. [5]

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(b) Find the magnitude and direction of the velocity of P one second before it strikes the plane. [4]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

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Cambridge Assessment International Education

Cambridge International AS & A Level

FURTHER MATHEMATICS **9231/32**

Paper 3 Further Mechanics **May/June 2021**

MARK SCHEME

Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2021 series for most Cambridge IGCSE™, Cambridge International A and AS Level components and some Cambridge O Level components.

This document consists of **13** printed pages.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	$\frac{\mathrm{d}v}{\sqrt{v}} = -\frac{10\mathrm{d}t}{(t+1)^2}$	**M1**	Separate variables.
	$2\sqrt{v} = \frac{10}{t+1} + A$	**M1 A1**	Attempt to integrate.
	$t = 0, \quad v = 25, \quad A = 0$	**M1**	Use correct initial condition.
	$v = \frac{25}{(t+1)^2}$	**A1**	CAO
		5	

Question	Answer	Marks	Guidance
2	$\uparrow\ R\cos\theta = mg$	B1	
	$\rightarrow\ R\sin\theta = \frac{mv^2}{r}$	B1	
	$r = a\sin\theta$	B1	
	$8\cos\theta = 3\left(1-(\cos\theta)^2\right)$	M1	Quadratic equation in $\cos\theta$.
	$\cos\theta = \frac{1}{3}$	A1	
	$x = \frac{2}{3}a$	A1	
		6	

Question	Answer	Marks	Guidance
3(a)	Use Hooke's Law: $4mg = \dfrac{kmg(x-a)}{a}$ leading to $k = \dfrac{4a}{x-a}$	**B1**	AG. Shown convincingly.
		1	
3(b)	Gain in KE + gain in EPE = loss in GPE	**B1**	One correct EPE term seen.
	$\frac{1}{2}\times 6m \times \frac{ga}{9} + \frac{1}{2}\frac{kmg}{a}\left(\left(x+\frac{a}{3}-a\right)^2 - (x-a)^2\right) = 6mg\times\frac{a}{3}$	**M1** **A1**	All 3 types of energy included in energy equation. All terms correct.
	Simplify and substitute for k from part **(a)**	**M1**	
	Obtain linear equation in x and a	**M1**	
	$x = \frac{5}{3}a$	**A1**	($k = 6$)
		6	

Question	Answer	Marks	Guidance
4(a)	<table><tr><td></td><td>Volume</td><td>Centre of mass from base of cylinder</td></tr><tr><td>Cone</td><td>$\frac{1}{3}\pi r^2 kh$</td><td>$h+\frac{kh}{4}$</td></tr><tr><td>Cylinder</td><td>$\pi r^2 h$</td><td>$\frac{h}{2}$</td></tr><tr><td>Combined</td><td>$\pi r^2 h\left(\frac{1}{3}k+1\right)$</td><td>$\bar{x}$</td></tr></table>	M1	
	Take moments about base: $\pi r^2 h\left(\frac{1}{3}k+1\right)\bar{x}=\frac{1}{3}\pi r^2 kh\left(h+\frac{kh}{4}\right)+\pi r^2 h\,\frac{h}{2}$	A1 A1	2 terms correct. All terms correct.
	$\bar{x}=\frac{h\left(k^2+4k+6\right)}{4(3+k)}$	A1	AG. Shown convincingly.
		4	
4(b)	$\tan\theta=\frac{r}{\bar{x}}$	M1	
	$\frac{4}{3}=\frac{6h(k+3)}{h\left(k^2+4k+6\right)}$ $2k^2-k-15=0$	M1	Equate to $\frac{4}{3}$ and simplify to quadratic.
	$k=3$	A1	CAO. No other solutions.
		3	

Question	Answer	Marks	Guidance
5	$T_A - mg\cos\theta = \frac{mu^2}{a}$	**B1**	
	$T_B + mg\cos\theta = \frac{mag}{a}$	**B1**	
	$T_A = 7T_B$ so $mg\cos\theta + \frac{mu^2}{a} = 7\left(-mg\cos\theta + \frac{mag}{a}\right)$ $u^2 = ag(7 - 8\cos\theta)$	**M1**	Use given relationship and combine.
	Energy: $\frac{1}{2}mu^2 - \frac{1}{2}mag = mg(a\cos\theta + a\cos\theta)$ So $u^2 = ag(4\cos\theta + 1)$	**M1 A1**	Energy equation.
	Equate expressions for u^2	**M1**	
	$\cos\theta = \frac{1}{2}$, $\theta = 60°$	**A1**	CAO
	$u = \sqrt{3ga}$	**A1**	CAO
		8	

Question	Answer	Marks	Guidance
6(a)	Along line of centres, speeds v_1 and v_2 $mv_1 + mv_2 = mu\cos\alpha - mu\cos\beta$	**M1**	Momentum (condone missing masses).
	$v_2 - v_1 = eu(\cos\beta + \cos\alpha)$	**M1**	Restitution.
	Both correct, masses seen.	**A1**	
	$v_1 = 0$ so A has no speed along line of centres: moves perpendicular to line of centres	**A1**	AG.
		4	
6(b)	$(v_2 = \frac{1}{2}u\cos\alpha = u\cos\beta\ )$ KE of B after collision is $\frac{1}{2}m\left(v_2{}^2 + (u\sin\beta)^2\right)$ KE of A after collision = $\frac{1}{2}m(u\sin\alpha)^2$	**M1**	Both components.
	Add both KEs and equate to $\frac{3}{4}mu^2$	**M1**	
	Simplify to equation in $\sin\alpha$	**M1**	
	$\sin\alpha = \frac{1}{\sqrt{2}}$, $\alpha = 45°$	**A1**	
		4	

Question	Answer	Marks	Guidance
7(a)	At greatest height $0 = 100\sin\theta - gt$	**M1**	
	$t = 8$	**A1**	
	Therefore times at height H are $t = 3$ (and $t = 13$)	**B1**	
	Substitute into $H = 100\sin\theta t - \frac{1}{2}gt^2$	**M1**	
	$H = 195$	**A1**	
	Alternative method to question **7(a)**		
	$\uparrow H = 100\sin\theta t - \frac{1}{2}gt^2$	**M1**	
	And $H = 100\sin\theta(t+10) - \frac{1}{2}g(t+10)^2$	**A1**	
	Subtract: $1000\sin\theta = \frac{1}{2}g(20t+100)$	**M1**	
	$t = 3$	**B1**	
	$H = 195$	**A1**	

Question	Answer	Marks	Guidance
7(a)	Alternative method to question **7(a)**		
	$\uparrow H = 100\sin\theta t - \frac{1}{2}gt^2$	**B1**	
	Difference between roots = $\dfrac{\sqrt{(100\sin\theta)^2 - 2gH}}{\frac{1}{2}g}$	**M1 A1**	
	Equate to 10 and rearrange to find H	**M1**	
	$H = 195$	**A1**	
		5	
7(b)	Time to required point = 15 s	**B1**	
	$\uparrow v = 100\sin\theta - 10\times 15 (= -70)$ $\rightarrow v = 100\cos\theta = 60$	**B1**	Both components.
	Magnitude = 92.2	**B1**	
	Angle below horizontal = $\tan^{-1}(70/60) = 49.4°$	**B1**	
		4	

Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

6007163164

FURTHER MATHEMATICS **9231/33**

Paper 3 Further Mechanics **May/June 2021**

1 hour 30 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
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- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.
- Where a numerical value for the acceleration due to gravity (g) is needed, use $10\,\text{m}\,\text{s}^{-2}$.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Any blank pages are indicated.

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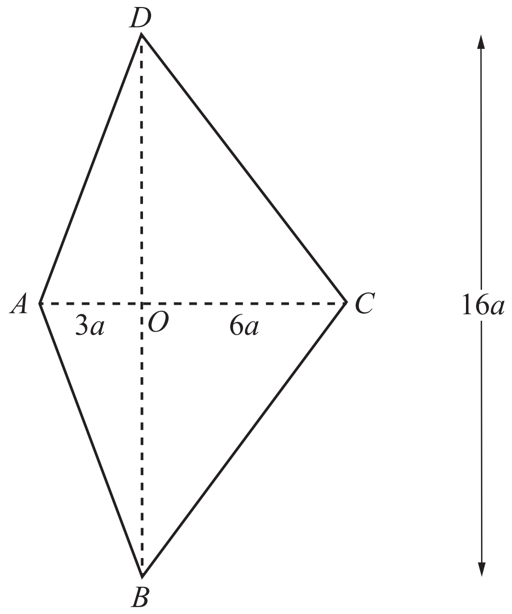

A uniform lamina $ABCD$ consists of two isosceles triangles ABD and BCD. The diagonals of $ABCD$ meet at the point O. The length of AO is $3a$, the length of OC is $6a$ and the length of BD is $16a$ (see diagram).

Find the distance of the centre of mass of the lamina from DB. [3]

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2 One end of a light elastic string of natural length 0.8 m and modulus of elasticity 36 N is attached to a fixed point O on a smooth plane. The plane is inclined at an angle α to the horizontal, where $\sin\alpha = \frac{3}{5}$. A particle P of mass 2 kg is attached to the other end of the string. The string lies along a line of greatest slope of the plane with the particle below the level of O. The particle is projected with speed $\sqrt{2}\,\mathrm{m\,s^{-1}}$ directly down the plane from the position where OP is equal to the natural length of the string.

Find the maximum extension of the string during the subsequent motion. [5]

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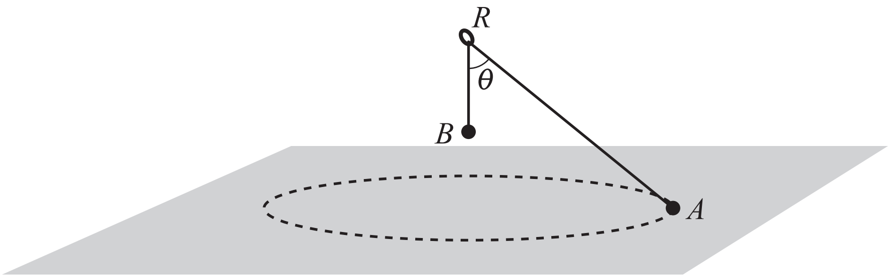

Particles A and B, of masses $3m$ and m respectively, are connected by a light inextensible string of length a that passes through a fixed smooth ring R. Particle B hangs in equilibrium vertically below the ring. Particle A moves in horizontal circles on a smooth horizontal surface with speed $\frac{2}{5}\sqrt{ga}$. The angle between AR and BR is θ (see diagram). The normal reaction between A and the surface is $\frac{12}{5}mg$.

(a) Find $\cos\theta$. [3]

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(b) Find, in terms of a, the distance of B below the ring. [3]

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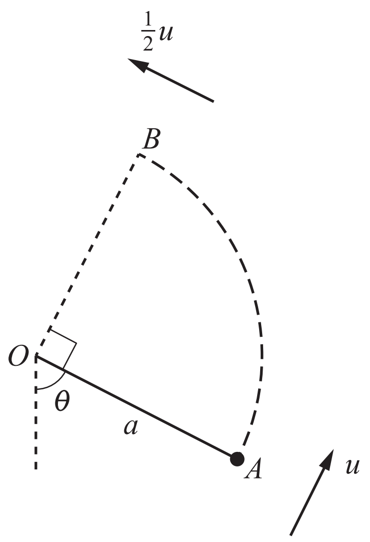

A particle of mass m is attached to one end of a light inextensible string of length a. The other end of the string is attached to a fixed point O. The particle is initially held with the string taut at the point A, where OA makes an angle θ with the downward vertical through O. The particle is then projected with speed u perpendicular to OA and begins to move upwards in part of a vertical circle. The string goes slack when the particle is at the point B where angle AOB is a right angle. The speed of the particle when it is at B is $\frac{1}{2}u$ (see diagram).

Find the tension in the string at A, giving your answer in terms of m and g. [8]

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5 A particle P of mass m kg is projected vertically upwards from a point O, with speed $20\,\mathrm{m\,s^{-1}}$, and moves under gravity. There is a resistive force of magnitude $2mv$ N, where $v\,\mathrm{m\,s^{-1}}$ is the speed of P at time t s after projection.

(a) Find an expression for v in terms of t, while P is moving upwards. [6]

The displacement of P from O is x m at time t s.

(b) Find an expression for x in terms of t, while P is moving upwards. [2]

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(c) Find, correct to 3 significant figures, the greatest height above O reached by P. [2]

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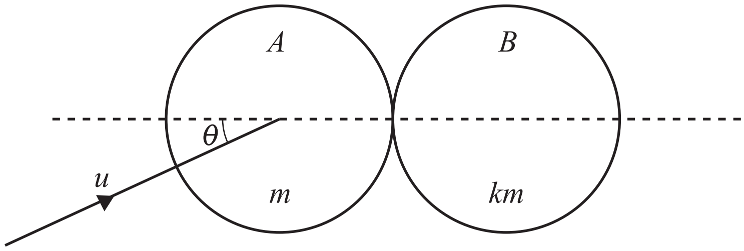

Two uniform smooth spheres A and B of equal radii have masses m and km respectively. Sphere A is moving with speed u on a smooth horizontal surface when it collides with sphere B which is at rest. Immediately before the collision, A's direction of motion makes an angle θ with the line of centres (see diagram). The coefficient of restitution between the spheres is $\frac{1}{3}$.

(a) Show that the speed of B after the collision is $\dfrac{4u\cos\theta}{3(1+k)}$. [3]

70% of the total kinetic energy of the spheres is lost as a result of the collision.

(b) Given that $\tan\theta = \frac{1}{3}$, find the value of k. [6]

7 A particle P is projected with speed u at an angle θ above the horizontal from a point O on a horizontal plane and moves freely under gravity. The horizontal and vertical displacements of P from O at a subsequent time t are denoted by x and y respectively.

(a) Use the equation of the trajectory given in the List of formulae (MF19), together with the condition $y = 0$, to establish an expression for the range R in terms of u, θ and g. [2]

(b) Deduce an expression for the maximum height H, in terms of u, θ and g. [2]

It is given that $R = \dfrac{4H}{\sqrt{3}}$.

(c) Show that $\theta = 60°$. [1]

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It is given also that $u = \sqrt{40}\,\mathrm{m\,s^{-1}}$.

(d) Find, by differentiating the equation of the trajectory or otherwise, the set of values of x for which the direction of motion makes an angle of less than 45° with the horizontal. [4]

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Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

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Cambridge Assessment International Education

Cambridge International AS & A Level

FURTHER MATHEMATICS **9231/33**

Paper 3 Further Mechanics **May/June 2021**

MARK SCHEME

Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2021 series for most Cambridge IGCSE™, Cambridge International A and AS Level components and some Cambridge O Level components.

This document consists of **16** printed pages.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:
Marks must be awarded in line with:
- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:
Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:
Marks must be awarded **positively**:
- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:
Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

<table>
<tr><th>Question</th><th>Answer</th><th>Marks</th><th>Guidance</th></tr>
<tr><td>1</td><td>
<table>
<tr><th></th><th>Area</th><th>Centre of mass from DB</th></tr>
<tr><td>ABD</td><td>$24a^2$</td><td>$-a$</td></tr>
<tr><td>BCD</td><td>$48a^2$</td><td>$2a$</td></tr>
<tr><td>Combined</td><td>$72a^2$</td><td>$\bar{x}$</td></tr>
</table>
</td><td>B1</td><td>All distances correct.

ABCD can be split in other ways, for example ADC and ABC.</td></tr>
<tr><td></td><td>Taking moments about DB:
$72a^2\bar{x} = 24a^2 \times -a + 48a^2 \times 2a$
OR
Taking moments about A:
$72a^2\bar{x} = 24a^2 \times 2a + 48a^2 \times 5a$
OR
Taking moments about G:
$24a^2(\bar{x}+a) = 48a^2 \times (2a-\bar{x})$</td><td>M1</td><td>Moments equation with masses in correct ratio.</td></tr>
<tr><td></td><td>$\bar{x} = a$</td><td>A1</td><td>CWO</td></tr>
<tr><td></td><td colspan="3">Alternative method for question 1</td></tr>
<tr><td></td><td>ADC: distance of centre of mass from BD = $\frac{6a-3a}{3} = a$

ABC: distance of centre of mass from BD = $\frac{6a-3a}{3} = a$</td><td>B1</td><td>One calculation.</td></tr>
<tr><td></td><td>Second calculation or statement about symmetry</td><td>M1</td><td></td></tr>
<tr><td></td><td>$\bar{x} = a$</td><td>A1</td><td></td></tr>
<tr><td></td><td></td><td>3</td><td></td></tr>
</table>

Question	Answer	Marks	Guidance
2	$\frac{1}{2}\times\frac{36x^2}{0.8}$ $(= 22.5x^2)$	**B1**	EPE correct.
	Loss in GPE + loss in KE = gain in EPE $x\,mg\sin\alpha+\frac{1}{2}m\times 2=\frac{1}{2}\times\frac{36x^2}{0.8}$	***M1**	Energy equation with only GPE, EPE and KE terms, allow sign errors, allow missing g for M1 only, weight must be resolved (allow sin or cos).
	All terms correct	**A1**	
	$\left(\frac{6}{5}x+\frac{1}{5}=\frac{9}{4}x^2\right)$ [leading to $45x^2-24x-4=0$]	**DM1**	Simplify to 3-term quadratic and attempt to solve.
	$(3x-2)(15x+2)=0$ $x=\frac{2}{3}$ only	**A1**	
		5	

Question	Answer	Marks	Guidance
3(a)	For B: $T = mg$	**B1**	May be embedded.
	For A: $R + T\cos\theta = 3mg$	**M1**	All 3 terms, allow sign errors, allow sin/cos mix.
	Use given R to obtain $\cos\theta = \frac{3}{5}$	**A1**	
		3	
3(b)	$T\sin\theta = \frac{3mv^2}{r}$	**M1**	May be seen in part **(a)**, allow sin/cos mix.
	$r = AR\sin\theta$	**B1**	Or equivalent.
	[Combine to give $AR = \frac{3a}{4}$, so] $BR = \frac{1}{4}a$	**A1**	
		3	

Question	Answer	Marks	Guidance
4	$\frac{1}{2}mu^2 - \frac{1}{2}m\left(\frac{u}{2}\right)^2 = mg(a\cos\theta + a\sin\theta)$	*M1	Energy equation, with 2 KE terms and a two-part GPE term, allow cos/sin mix.
	$\frac{3}{4}u^2 = 2ag(\cos\theta + \sin\theta)$	A1	
	At B, tension in string is zero, so $mg\sin\theta = \frac{m\left(\frac{u}{2}\right)^2}{a}$ $(u^2 = 4ag\sin\theta)$	B1	N2L
	Eliminate u^2	DM1	
	$\tan\theta = 2$ OE	A1	
	At A, $T - mg\cos\theta = \frac{mu^2}{a}$	B1	N2L
	$T = \frac{9\sqrt{5}}{5}mg \quad (= 4.02mg)$	M1 A1	Substitute to find T.
	Alternative method for question 4		
	$\frac{1}{2}mu^2 - \frac{1}{2}m\left(\frac{u}{2}\right)^2 = mg(a\cos\theta + a\sin\theta)$	*M1	Energy equation, with 2 KE terms and a two-part GPE term, allow cos/sin mix.
	$\frac{3}{4}u^2 = 2ag(\cos\theta + \sin\theta)$	A1	

Question	Answer	Marks	Guidance
4	At B, tension in string is zero, so $mg\sin\theta = \dfrac{m\left(\frac{u}{2}\right)^2}{a}$ $(u^2 = 4ag\sin\theta)$	B1	N2L
	Eliminate θ: $u^2 = \dfrac{8ag\sqrt{5}}{5}$	DM1 A1	
	At A, $T - mg\cos\theta = \dfrac{mu^2}{a}$	B1	N2L
	$T = \dfrac{9\sqrt{5}}{5}mg \quad (= 4.02mg)$	M1 A1	Substitute to find T.
		8	

Question	Answer	Marks	Guidance
5(a)	$m\frac{dv}{dt} = -mg - 2mv$	**B1**	Use of SUVAT implies 0 marks. N2L, must include m.
	$\ln(5+v) = -2t(+A)$	**M1**	Separate variables and integrate 3-term N2L, condone omission of constant.
	$\ln(5+v) = -2t + A$	**A1 FT**	FT only sign error in N2L.
	$t = 0, v = 20, \quad A = \ln 25$	**M1**	Use correct initial condition.
	$2t = \ln\left(\frac{25}{5+v}\right)$, $e^{2t} = \frac{25}{5+v}$	**M1**	Remove all logs.
	$v = 25e^{-2t} - 5$	**A1**	
	Alternative method for question 5(a)		
	$m\frac{dv}{dt} = -mg - 2mv$	**B1**	N2L, must include m.
	$\frac{dv}{dt} + 2v = -g$: Integrating factor = e^{2t}	**M1**	
	$\frac{d\left(ve^{2t}\right)}{dt} = -ge^{2t}$, $\quad ve^{2t} = -\frac{g}{2}e^{2t}(+A)$	**M1**	Integrate both sides, condone omission of constant.
	$ve^{2t} = -\frac{g}{2}e^{2t} + \text{A}$	**A1 FT**	FT only sign error in N2L.

Question	Answer	Marks	Guidance
5(a)	$t=0, v=20, \quad A=25$	**M1**	Use correct initial condition.
	$ve^{2t}=-\frac{g}{2}e^{2t}+25, \quad v=25e^{-2t}-5$	**A1**	
		6	
5(b)	$x=-\frac{25}{2}e^{-2t}-5t(+B)$	**M1**	Use of SUVAT implies 0 marks. Integrate their expression from part **(a)**.
	$t=0, x=0, \quad B=\frac{25}{2}$ $x=\frac{25}{2}\left(1-e^{-2t}\right)-5t$	**A1 FT**	FT only expressions of the form $v=Pe^{kt}+Q$ for P, Q non-zero.
		2	
5(c)	Greatest height when $v=0$, so $t=0.8047\ldots$or $\frac{1}{2}\ln 5$	**M1**	Use of SUVAT in part **(a)** or part **(b)** implies 0 marks. Find value of t, may be embedded.
	$x=5.98$ m	**A1**	CWO
		2	

Question	Answer	Marks	Guidance
6(a)	Let velocities of A and B along line of centres after collision be v_1 and v_2. $mv_1 + kmv_2 = mu\cos\theta$.	**M1**	Momentum, must include m, allow cos/sin mix.
	$v_2 - v_1 = \frac{1}{3}u\cos\theta$	**M1**	Restitution, consistent signs, correct way up.
	Solve: $v_2 = \frac{4u\cos\theta}{3(1+k)}$	**A1**	AG shown convincingly.
		3	
6(b)	$v_1 = \frac{(3-k)u\cos\theta}{3(1+k)}$	**B1**	Or equivalent, may be unsimplified.
	Use velocity of A with both components.	**B1**	$v_1^2 + (u\sin\theta)^2$ seen.
	$\frac{1}{2}kmv_2^2 + \frac{1}{2}m\left(v_1^2 + (u\sin\theta)^2\right) = \frac{3}{10}\times\frac{1}{2}mu^2$	**M1**	KE after = 30% KE before (all terms present). M0 if incorrect masses.
	Substitute from part **(a)** and for θ.	**M1**	Eliminate trigonometric terms, must be KE equation, in terms of k only.
	$(3-k)^2 + 16k = 2(1+k)^2$, $k^2 - 6k - 7 = 0$	**M1**	Obtain simplified quadratic equation in k .
	$k = 7$	**A1**	
		6	

Question	Answer	Marks	Guidance
7(a)	$y = 0$ in trajectory equation: $R\tan\theta - g\dfrac{R^2}{2u^2(\cos\theta)^2} = 0$	M1	
	$(R =)\dfrac{2u^2\sin\theta\cos\theta}{g}$ only	A1	Any equivalent single term expression, for example: $\dfrac{u^2\sin 2\theta}{g}$, $\dfrac{2u^2\tan\theta}{g\sec^2\theta}$, at least one intermediate line of working, not just quoting a result. SC B1 using SUVAT.
		2	
7(b)	$x = their\dfrac{u^2\sin\theta\cos\theta}{g}$ and substitute in trajectory equation.	M1	Or use SUVAT.
	$H = \dfrac{u^2(\sin\theta)^2}{2g}$	A1	Single term.
		2	
7(c)	Use $R = \dfrac{4H}{\sqrt{3}}$ and simplify: $\tan\theta = \sqrt{3},\ \theta = 60°$	B1	AG
		1	

Question	Answer	Marks	Guidance
7(d)	$\dfrac{dy}{dx}=\tan\theta-\dfrac{gx}{u^2(\cos\theta)^2}$	M1	Differentiate with respect to *x*.
	$\tan\theta-\dfrac{x}{4(\cos\theta)^2}=\pm1$ used	M1	Use $\dfrac{dy}{dx}=\pm1$ as limiting case.
	$x=\sqrt{3}+1$, $x=\sqrt{3}-1$	A1	
	$\sqrt{3}-1<x<\sqrt{3}+1$	A1	Strict inequality, exact values.
	Alternative method for question 7(d)		
	$y=\sqrt{3}x-\dfrac{1}{2}x^2$, $\dfrac{dy}{dx}=\sqrt{3}-x$	M1	Differentiate with respect to *x*.
	$\dfrac{dy}{dx}=\pm1$ used	M1	Use $\dfrac{dy}{dx}=\pm1$ as limiting case.
	$x=\sqrt{3}+1$, $x=\sqrt{3}-1$	A1	
	$\sqrt{3}-1<x<\sqrt{3}+1$	A1	Strict inequality, exact values.

Question	Answer	Marks	Guidance
7(d)	**Alternative method for question 7(d)**		
	When moving at 45° to horizontal, $v_x = \pm v_y$	**M1**	Used, both cases considered.
	$v_x = \sqrt{40}\cos\theta,\ v_y = \sqrt{40}\sin\theta - 10t$ $t = \frac{1}{10}\left(\sqrt{30} - \sqrt{10}\right)$, $t = \frac{1}{10}\left(\sqrt{30} + \sqrt{10}\right)$	**M1**	
	$x = \sqrt{3} + 1$, $x = \sqrt{3} - 1$	**A1**	
	$\sqrt{3} - 1 < x < \sqrt{3} + 1$	**A1**	Strict inequality, exact values.
		4	

Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME			
CENTRE NUMBER		CANDIDATE NUMBER	

4618189778

FURTHER MATHEMATICS **9231/31**

Paper 3 Further Mechanics **October/November 2020**

1 hour 30 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.
- Where a numerical value for the acceleration due to gravity (g) is needed, use $10\,\mathrm{m\,s^{-2}}$.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

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1 A particle P of mass m is placed on a fixed smooth plane which is inclined at an angle θ to the horizontal. A light spring, of natural length a and modulus of elasticity $3mg$, has one end attached to P and the other end attached to a fixed point O at the top of the plane. The spring lies along a line of greatest slope of the plane. The system is released from rest with the spring at its natural length.

Find, in terms of a and θ, an expression for the greatest extension of the spring in the subsequent motion. [3]

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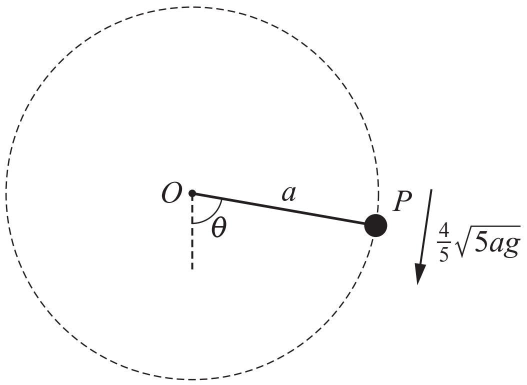

A particle P is attached to one end of a light inextensible string of length a. The other end of the string is attached to a fixed point O. The particle P is held with the string taut and making an angle θ with the downward vertical. The particle P is then projected with speed $\frac{4}{5}\sqrt{5ag}$ perpendicular to the string and just completes a vertical circle (see diagram).

Find the value of $\cos\theta$. [5]

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3 One end of a light elastic string, of natural length a and modulus of elasticity $4mg$, is attached to a fixed point O. The other end of the string is attached to a particle of mass m. The particle moves in a horizontal circle with a constant angular speed $\sqrt{\frac{g}{a}}$ with the string inclined at an angle θ to the downward vertical through O. The length of the string during this motion is $(k+1)a$.

(a) Find the value of k. [4]

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(b) Find the value of $\cos\theta$. [2]

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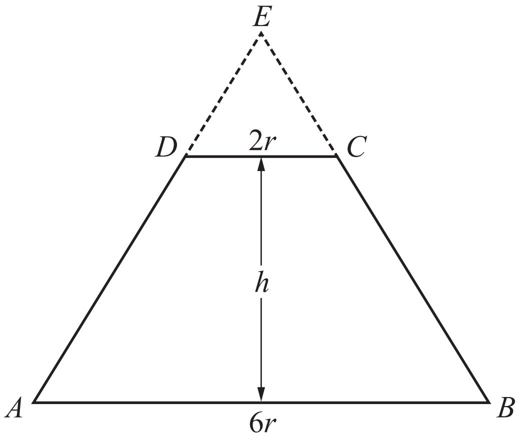

The diagram shows the cross-section *ABCD* of a uniform solid object which is formed by removing a cone with cross-section *DCE* from the top of a larger cone with cross-section *ABE*. The perpendicular distance between *AB* and *DC* is h, the diameter *AB* is $6r$ and the diameter *DC* is $2r$.

(a) Find an expression, in terms of h, for the distance of the centre of mass of the solid object from *AB*. [4]

The object is freely suspended from the point B and hangs in equilibrium. The angle between AB and the downward vertical through B is θ.

(b) Given that $h = \frac{13}{4}r$, find the value of $\tan\theta$. [2]

5 A particle P is projected with speed u at an angle α above the horizontal from a point O on a horizontal plane and moves freely under gravity. The horizontal and vertical displacements of P from O at a subsequent time t are denoted by x and y respectively.

(a) Derive the equation of the trajectory of P in the form

$$y = x\tan\alpha - \frac{gx^2}{2u^2}\sec^2\alpha.$$ [3]

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The point Q is the highest point on the trajectory of P in the case where $\alpha = 45°$.

(b) Show that the x-coordinate of Q is $\frac{u^2}{2g}$. [3]

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(c) Find the other value of α for which P would pass through the point Q. [4]

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6 Two smooth spheres A and B have equal radii and masses m and $2m$ respectively. Sphere B is at rest on a smooth horizontal floor. Sphere A is moving on the floor with velocity u and collides directly with B. The coefficient of restitution between the spheres is e.

(a) Find, in terms of u and e, the velocities of A and B after the collision. [3]

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Subsequently, B collides with a fixed vertical wall which makes an angle θ with the direction of motion of B, where $\tan\theta = \frac{3}{4}$.

The coefficient of restitution between B and the wall is $\frac{2}{3}$. Immediately after B collides with the wall, the kinetic energy of A is $\frac{5}{32}$ of the kinetic energy of B.

(b) Find the possible values of e. [7]

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7 A particle P moving in a straight line has displacement x m from a fixed point O on the line at time t s. The acceleration of P, in m s^{-2}, is given by $\frac{200}{x^2} - \frac{100}{x^3}$ for $x > 0$. When $t = 0$, $x = 1$ and P has velocity $10\,\text{m s}^{-1}$ directed towards O.

(a) Show that the velocity v m s^{-1} of P is given by $v = \frac{10(1-2x)}{x}$. [5]

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(b) Show that x and t are related by the equation $e^{-40t} = (2x-1)e^{2x-2}$ and deduce what happens to x as t becomes large. [5]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

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Cambridge Assessment International Education

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FURTHER MATHEMATICS **9231/31**

Paper 3 Further Mechanics 31 **October/November 2020**

MARK SCHEME

Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

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Generic Marking Principles

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GENERIC MARKING PRINCIPLE 1: Marks must be awarded in line with: • the specific content of the mark scheme or the generic level descriptors for the question • the specific skills defined in the mark scheme or in the generic level descriptors for the question • the standard of response required by a candidate as exemplified by the standardisation scripts.
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GENERIC MARKING PRINCIPLE 3: Marks must be awarded **positively**: • marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate • marks are awarded when candidates clearly demonstrate what they know and can do • marks are not deducted for errors • marks are not deducted for omissions • answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.
GENERIC MARKING PRINCIPLE 4: Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.
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Mathematics Specific Marking Principles	
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Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	Gain in EPE = $\frac{1}{2}.\frac{3mgx^2}{a}$	B1	EPE gain.
	Loss in GPE = $mgx\sin\theta$ Equate	M1	Equate energies
	$x = \frac{2}{3}a\sin\theta$	A1	Using forces scores B0M0A0
		3	

Question	Answer	Marks	Guidance
2	At top, tension = 0, so $mg = \frac{mv^2}{a}$ $(v^2 = ag)$	B1	
	$\frac{1}{2}mv^2 = \frac{1}{2}mu^2 - mga(1+\cos\theta)$	M1 A1	Energy equation
	Substitute for u and v: $ag = \frac{16}{25}.5ag - 2ag(1+\cos\theta)$	M1	Eliminate
	$\cos\theta = \frac{1}{10}$	A1	
		5	

Question	Answer	Marks	Guidance
3(a)	$T = 4mg.\frac{ka}{a}$	B1	Use Hooke's law
	$T\sin\theta = \left(\frac{mrg}{a}\right) = m(k+1)a\sin\theta.\frac{g}{a}$	M1	N2L horizontally. Must see T and k.
	$T = mg(k+1)$	A1	
	Equate: $k = \frac{1}{3}$	A1	
		4	
3(b)	$\uparrow T\cos\theta = mg$	M1	
	$(T = \frac{4}{3}mg)$ $\cos\theta = \frac{mg}{\frac{4}{3}mg} = \frac{3}{4}$	A1	
		2	

<table>
<tr><th>Question</th><th>Answer</th><th>Marks</th><th>Guidance</th></tr>
<tr><td rowspan="4">4(a)</td><td>

	Volume	Centre of mass from AB
Small cone	$\frac{1}{3}\pi r^2 . \frac{h}{2}$	$h + \frac{1}{4} . \frac{h}{2}\left(= \frac{9h}{8}\right)$
Large cone	$\frac{1}{3}\pi(3r)^2 . \frac{3h}{2}$	$\frac{1}{4} . \frac{3h}{2}\left(= \frac{3h}{8}\right)$
Object	$\frac{26}{6}\pi(r)^2 h$	$\bar{x}$

</td><td>B1</td><td>For 9h/8 or 3h/8 (unsimplified)</td></tr>
<tr><td>Take moments about AB
$\frac{13}{3}\pi r^2 h.\bar{x} = \frac{27}{6}\pi r^2 h.\frac{3h}{8} - \frac{1}{6}\pi r^2 h.\frac{9h}{8}$</td><td>M1 A1</td><td>Moments equation: Allow use of relative masses 1, 26, 27</td></tr>
<tr><td>$\bar{x} = \frac{9h}{26}$</td><td>A1</td><td></td></tr>
<tr><td></td><td>4</td><td></td></tr>
<tr><td rowspan="3">4(b)</td><td>$\tan\theta = \frac{\bar{x}}{3r}$</td><td>M1</td><td></td></tr>
<tr><td>$(= \frac{3h}{26r})$ Use $h = \frac{13}{4}r$
$\tan\theta = \frac{3}{8}$</td><td>A1</td><td></td></tr>
<tr><td></td><td>2</td><td></td></tr>
</table>

Question	Answer	Marks	Guidance
5(a)	$\rightarrow x = u\cos\alpha t$ $\uparrow y = u\sin\alpha t - \frac{1}{2}gt^2$	B1	Both
	Eliminate t: $y = u\sin\alpha.\frac{x}{u\cos\alpha} - \frac{1}{2}g\left(\frac{x}{u\cos\alpha}\right)^2$	M1	Eliminate
	$y = x\tan\alpha - \frac{gx^2}{2u^2}\sec^2\alpha$	A1	AG
		3	
5(b)	Greatest height $= \frac{(u\sin\alpha)^2}{2g} = \frac{u^2}{4g}$	M1 A1	Accept alternative methods, for example differentiate expression in **(a)** and equate to 0.
	$t = u\sin 45/\text{g}$ so $d = u\cos 45.u\sin 45/\text{g} = \frac{u^2}{2g}$	A1	AG
		3	

Question	Answer	Marks	Guidance
5(c)	Use greatest height displacements in trajectory equation $\frac{u^2}{4g}=\frac{u^2}{2g}\tan\alpha-\frac{gu^4}{2u^2 4g^2}\sec^2\alpha$	M1	Use equation of trajectory (substitute coordinates of Q
	$u^2=2u^2\tan\alpha-\frac{u^2}{2}(1+\tan^2\alpha)$	M1	Use of $\sec^2\alpha=\left(1+\tan^2\alpha\right)$
	$\tan^2\alpha-4\tan\alpha+3=0$	M1	Obtain a three-term quadratic in $\tan\alpha$
	$\tan\alpha=1,\ 3$ so $\alpha=71.6°$	A1	Both solutions needed
		4	

Question	Answer	Marks	Guidance
6(a)	$mu = mw + 2mv$	**B1**	Momentum equation (with m)
	$v - w = eu$	**B1**	Restitution with consistent signs
	$v = \frac{u}{3}(e+1)$ $w = \frac{u}{3}(1-2e)$	**B1**	Both correct.
		3	

Question	Answer	Marks	Guidance
6(b)	Perpendicular to plane: $y = ev\sin\theta$ Parallel to plane: $x = v\cos\theta$	**B1**	Both
	Speed of $B = \sqrt{x^2+y^2} = \sqrt{v^2\left(\left(\frac{4}{5}\right)^2+\left(\frac{2}{3}.\frac{3}{5}\right)^2\right)}$ $\left(= \frac{2}{\sqrt{5}}v\right)$	**M1**	Speed of B
	KE of $B = \frac{1}{2}.2m\frac{4}{5}.\frac{u^2}{9}(e+1)^2$	**M1**	KE of B in terms of u. $\frac{1}{2}$ and $2m$ needed
	KE of $A = \frac{1}{2}.m.\frac{u^2}{9}(1-2e)^2$ So $\frac{1}{2}.m.\frac{u^2}{9}(1-2e)^2 = \frac{5}{32}.\frac{1}{2}.2m\frac{4}{5}.\frac{u^2}{9}(e+1)^2$	**M1 A1**	Relate the two KEs
	$4(1-2e)^2 = (e+1)^2$ or $15e^2 - 18e + 3 = 0$	**M1**	Rearrange and simplify to quadratic
	$1+e = \pm 2(1-2e)$ $e = \frac{1}{5},\ 1$	**A1**	Both values
		7	

Question	Answer	Marks	Guidance
7(a)	$v\frac{\mathrm{d}v}{\mathrm{d}x}=-\frac{100}{x^3}+\frac{200}{x^2}$ $\frac{v^2}{2}=\frac{50}{x^2}-\frac{200}{x}+A$	**M1 A1**	Correct equation and attempt to integrate Correct
	$x=1, v=-10: \quad A=200$	**M1**	Use initial condition
	$v^2=\frac{100(2x-1)^2}{x^2}$	**M1**	Rearrange to find v^2
	$v=\pm\frac{10(2x-1)}{x}$ and take negative sign to meet initial condition, so $v=\frac{10(1-2x)}{x}$	**A1**	Convincingly shown (no mention of $\pm$ scores A0) AG
		5	

Question	Answer	Marks	Guidance
7(b)	$\frac{x\mathrm{d}x}{1-2x}=10\mathrm{d}t$ $\frac{1}{2}\left(\frac{1}{1-2x}-1\right)\mathrm{d}x=10\mathrm{d}t$ $-\frac{1}{4}\ln\lvert 1-2x\rvert-\frac{x}{2}=10t+B$	**M1 A1**	Rearrange and attempt to integrate
	$t=0, x=1: \quad B=-\frac{1}{2}$	**M1**	Use initial condition
	$2x-2=-40t-\ln(\lvert 1-2x\rvert)$ so $e^{-40t}=(2x-1)e^{2x-2}$	**A1**	Convincingly shown, working required AG
	For large values of t, $x\to\frac{1}{2}$	**B1**	CAO
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Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

8938495195

FURTHER MATHEMATICS **9231/32**

Paper 3 Further Mechanics **October/November 2020**

1 hour 30 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.
- Where a numerical value for the acceleration due to gravity (g) is needed, use $10\,\mathrm{m\,s^{-2}}$.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Blank pages are indicated.

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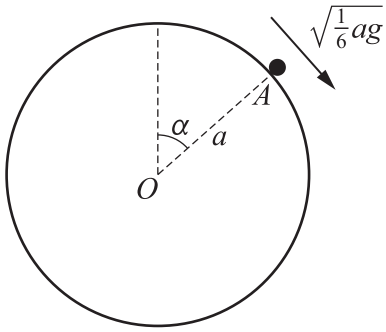

A fixed smooth solid sphere has centre O and radius a. A particle of mass m is projected downwards with speed $\sqrt{\frac{1}{6}ag}$ from the point A on the surface of the sphere, where OA makes an angle α with the upward vertical through O (see diagram). The particle moves in part of a vertical circle on the surface of the sphere. It loses contact with the sphere at the point B, where OB makes an angle β with the upward vertical through O.

Given that $\cos\alpha = \frac{2}{3}$, find the value of $\cos\beta$. [5]

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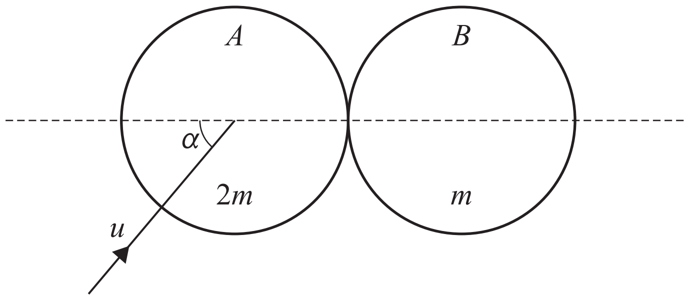

Two uniform smooth spheres A and B of equal radii have masses $2m$ and m respectively. Sphere B is at rest on a smooth horizontal surface. Sphere A is moving on the surface with speed u and collides with B. Immediately before the collision, the direction of motion of A makes an angle α with the line of centres of the spheres, where $\tan\alpha = \frac{4}{3}$ (see diagram). The coefficient of restitution between the spheres is $\frac{1}{3}$.

Find the speed of A after the collision. [5]

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3 An object consists of a uniform solid circular cone, of vertical height $4r$ and radius $3r$, and a uniform solid cylinder, of height $4r$ and radius $3r$. The circular base of the cone and one of the circular faces of the cylinder are joined together so that they coincide. The cone and the cylinder are made of the same material.

(a) Find the distance of the centre of mass of the object from the end of the cylinder that is not attached to the cone. [4]

(b) Show that the object can rest in equilibrium with the curved surface of the cone in contact with a horizontal surface. [3]

4 A particle P of mass m is moving in a horizontal circle with angular speed ω on the smooth inner surface of a hemispherical shell of radius r. The angle between the vertical and the normal reaction of the surface on P is θ.

(a) Show that $\cos\theta = \frac{g}{\omega^2 r}$. [3]

The plane of the circular motion is at a height x above the lowest point of the shell. When the angular speed is doubled, the plane of the motion is at a height $4x$ above the lowest point of the shell.

(b) Find x in terms of r. [4]

5 A particle P is projected with speed $u\,\mathrm{m\,s^{-1}}$ at an angle of θ above the horizontal from a point O on a horizontal plane and moves freely under gravity. The horizontal and vertical displacements of P from O at a subsequent time t s are denoted by x m and y m respectively.

(a) Starting from the equation of the trajectory given in the List of formulae (MF19), show that

$$y = x\tan\theta - \frac{gx^2}{2u^2}(1 + \tan^2\theta).$$ [1]

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When $\theta = \tan^{-1} 2$, P passes through the point with coordinates (10, 16).

(b) Show that there is no value of θ for which P can pass through the point with coordinates (18, 30). [6]

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6 One end of a light elastic string, of natural length a and modulus of elasticity k, is attached to a particle P of mass m. The other end of the string is attached to a fixed point Q. The particle P is projected vertically upwards from Q. When P is moving upwards and at a distance $\frac{4}{3}a$ directly above Q, it has a speed $\sqrt{2ga}$. At this point, its acceleration is $\frac{7}{3}g$ downwards.

Show that $k = 4mg$ and find in terms of a the greatest height above Q reached by P. [8]

7 A particle P of mass m kg moves in a horizontal straight line against a resistive force of magnitude mkv^2 N, where v m s^{-1} is the speed of P after it has moved a distance x m and k is a positive constant. The initial speed of P is u m s^{-1}.

(a) Show that $x = \frac{1}{k}\ln 2$ when $v = \frac{1}{2}u$. [4]

Beginning at the instant when the speed of P is $\frac{1}{2}u$, an additional force acts on P. This force has magnitude $\frac{5m}{v}$ N and acts in the direction of increasing x.

(b) Show that when the speed of P has increased again to $u \text{ m s}^{-1}$, the total distance travelled by P is given by an expression of the form

$$\frac{1}{3k}\ln\left(\frac{A-ku^3}{B-ku^3}\right),$$

stating the values of the constants A and B. [7]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

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FURTHER MATHEMATICS **9231/32**

Paper 3 Further Mechanics 32 **October/November 2020**

MARK SCHEME

Maximum Mark: 50

Published

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A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	At B, $mg\cos\beta = \frac{mv^2}{a}$: ($v^2 = ag\cos\beta$)	B1	
	$\frac{1}{2}mv^2 = \frac{1}{2}mu^2 + mga(\cos\alpha - \cos\beta)$	M1A1	Energy equation with 4 terms and correct dimensions $(v^2 = \frac{3ga}{2} - 2ag\cos\beta)$
	Substitute for u, $\cos\alpha$ and v: $ag\cos\beta = \frac{ag}{6} + 2ag\left(\frac{2}{3} - \cos\beta\right)$	M1	Eliminate to find $\cos\beta$
	$\cos\beta = \frac{1}{2}$	A1	
		5	

Question	Answer	Marks	Guidance
2	Speeds v and w after collision $2mv + mw = 2mu\cos\alpha$	M1	Momentum equation with m. Correct masses, allow sin instead of cos
	$w - v = eu\cos\alpha$	M1	Restitution, with consistent signs
	$v = \frac{1}{3}u\cos\alpha(2-e) = \frac{1}{3}u.\frac{3}{5}\left(2-\frac{1}{3}\right) = \frac{1}{3}u$	A1	
	Square of speed of A = $\left(\frac{1}{3}u\right)^2 + (u\sin\alpha)^2$	M1	Uses correct speed perpendicular to motion
	$= \left(\frac{1}{3}u\right)^2 + \left(\frac{4}{5}u\right)^2$ Speed = $\frac{13}{15}u$ $(= 0.867u)$	A1	
		5	

Question	Answer	Marks	Guidance
3(a)	<table><tr><td></td><td>Volume</td><td>Centre of mass from base</td></tr><tr><td>Cone</td><td>$\frac{1}{3}\pi(3r)^2 . 4r$</td><td>$4r+r$</td></tr><tr><td>Cylinder</td><td>$\pi(3r)^2 . 4r$</td><td>$2r$</td></tr><tr><td>Combined</td><td>$\frac{4}{3}\pi(3r)^2 . 4r$</td><td>$\bar{x}$</td></tr></table>	**B1**	Distances correct
	Taking moments about base of cylinder: $\bar{x}.\frac{4}{3}\pi(3r)^2 .4r = \frac{1}{3}\pi(3r)^2 .4r.5r + \pi(3r)^2 .4r.2r$	**M1 A1**	Moments equation
	$\bar{x} = \frac{11}{4}r$	**A1**	
		4	
3(b)	Condition: $OG\cos\theta < OA$ (where O is vertex of cone and OA is slant height of cone)	**B1**	Correct condition for equilibrium
	$\left(4r + \frac{5r}{4}\right)\times\frac{4}{5} < 5r$	**M1**	Expression in terms of r
	21 < 25 True	**A1**	Correct conclusion, with correct working
		3	

Question	Answer	Marks	Guidance
4(a)	$\uparrow\ N\cos\theta = mg$	**B1**	
	$\leftarrow\ N\sin\theta = mr\sin\theta\omega^2$	**B1**	
	$\cos\theta = \frac{mg}{N}$ so $\cos\theta = \frac{g}{\omega^2 r}$	**B1**	AG
		3	
4(b)	$\cos\theta = \frac{r-x}{r} = \frac{g}{\omega^2 r}$	**B1**	Using trig of situation: must involve x
	In new situation: $r - 4x = r \times \frac{g}{4\omega^2 r}$	**M1**	Using new situation with $4x$ and 2ω seen
	$r - x = 4(r - 4x)$	**M1**	Combining
	$x = \frac{1}{5}r$	**A1**	
		4	

Question	Answer	Marks	Guidance
5(a)	Quote trajectory equation from MF19 and use $\cos\theta = 1/\sec\theta$ $y = x\tan\theta - \frac{gx^2}{2u^2}(1+ \quad \tan^2\theta)$	B1	Must include step with $\sec^2\theta$ Allow derived from first principles AG
		1	
5(b)	$16 = 20 - \frac{10\times100}{2u^2}(1+4)$	M1	Substitute into result **(a)**
	$u^2 = 625, \quad (u = 25)$	A1	
	Use equation again: $30 = 18\tan\theta - \frac{10\times324}{2\times625}\left(1+(\tan\theta)^2\right)$	M1	
	$2.592(\tan\theta)^2) - 18\tan\theta + 32.592 = 0$	A1	3 term quadratic. Alternatives include: $54t^2 - 375t + 679 = 0$, $324t^2 - 2250t + 4074 = 0$
	Discriminant = $324 - 4\times2.592\times32.592 = -13.91$	M1	Discriminant for alternatives: −6039 and −217404
	As this is less than 0, no real solutions for θ	A1	CWO
		6	

Question	Answer	Marks	Guidance
6	$T + mg = m.\frac{7}{3}g$	M1	
	With $T = k\frac{\frac{a}{3}}{a}$ giving $k = 4mg$	A1	AG
	Let greatest height above Q be $\frac{4}{3}a + x$ Gain in GPE = mgx and Loss in KE = ½ $m.2ga$	B1	The length being found may be expressed as the total extension of the string or the greatest height above Q. GPE and KE
	Gain in EPE = $\frac{1}{2}.\frac{4mg}{a}\left(\left(x+\frac{a}{3}\right)^2 - \left(\frac{a}{3}\right)^2\right)$	B1	EPE Note: initial EPE = $\frac{2mga}{9}$
	$\frac{4mg}{2a}\left(x^2 + \frac{2ax}{3} + \frac{a^2}{9} - \frac{a^2}{9}\right) + mgx = mga$	M1 A1	Energy equation, correct number of terms
	$2x^2 + \frac{7ax}{3} - a^2 = 0$	M1	Simplify to quadratic
	$x = \frac{1}{3}a$ so greatest height is $\frac{5}{3}a$	A1	

Question	Answer	Marks	Guidance
7(a)	$mv\dfrac{dv}{dx} = -kmv^2$	B1	N2L, with m
	$\ln v = -kx + c$	M1	Separate variables and integrate
	$x = 0, v = u: \quad c = \ln u$	M1	Use initial condition
	$v = \dfrac{1}{2}u: \quad \ln\dfrac{1}{2} = -kx,$ $x = \dfrac{1}{k}\ln 2$	A1	AG
		4	
7(b)	$mv\dfrac{\mathrm{d}v}{\mathrm{d}x} = -mkv^2 + \dfrac{5m}{v}$	B1	N2L (allow missing m in this part)
	$\dfrac{v^2\mathrm{d}v}{5-kv^3} = \mathrm{d}x \; -\dfrac{1}{3k}\ln\left(5-kv^3\right) = x(+d)$	M1A1	Separate variables and integrate
	Using **(a)** $-\dfrac{1}{3k}\ln\left(5-kv^3\right) = x - \dfrac{1}{3k}\ln\left(5-ku^3\right) - \dfrac{1}{k}\ln 2$	M1M1	Use condition. M0 if $v = \dfrac{1}{2}u, x = 0$ used unless $\dfrac{1}{k}\ln 2$ is added on later Rearrange dependent on ln solution
	$x = \dfrac{1}{3k}\ln\left(\dfrac{40-ku^3}{5-ku^3}\right)$	M1A1	Use $v = u$
		7	

Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME						
CENTRE NUMBER					CANDIDATE NUMBER	

4073650141

FURTHER MATHEMATICS **9231/32**

Paper 3 Further Mechanics **May/June 2020**

1 hour 30 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.
- Where a numerical value for the acceleration due to gravity (g) is needed, use $10\,\text{m}\,\text{s}^{-2}$.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Blank pages are indicated.

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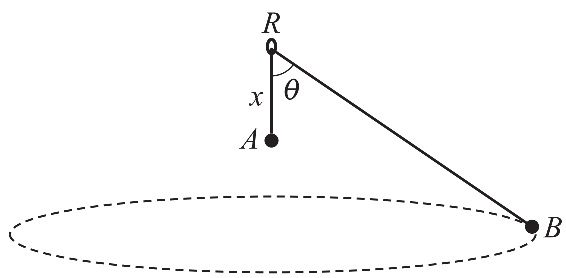
1 A particle P is projected with speed u at an angle of $30°$ above the horizontal from a point O on a horizontal plane and moves freely under gravity. The particle reaches its greatest height at time T after projection.

Find, in terms of u, the speed of P at time $\frac{2}{3}T$ after projection. [5]

2



A light inextensible string of length a is threaded through a fixed smooth ring R. One end of the string is attached to a particle A of mass $3m$. The other end of the string is attached to a particle B of mass m. The particle A hangs in equilibrium at a distance x vertically below the ring. The angle between AR and BR is θ (see diagram). The particle B moves in a horizontal circle with constant angular speed $2\sqrt{\frac{g}{a}}$.

Show that $\cos\theta = \frac{1}{3}$ and find x in terms of a. [5]

3 One end of a light elastic spring, of natural length a and modulus of elasticity $5mg$, is attached to a fixed point A. The other end of the spring is attached to a particle P of mass m. The spring hangs with P vertically below A. The particle P is released from rest in the position where the extension of the spring is $\frac{1}{2}a$.

(a) Show that the initial acceleration of P is $\frac{3}{2}g$ upwards. [3]

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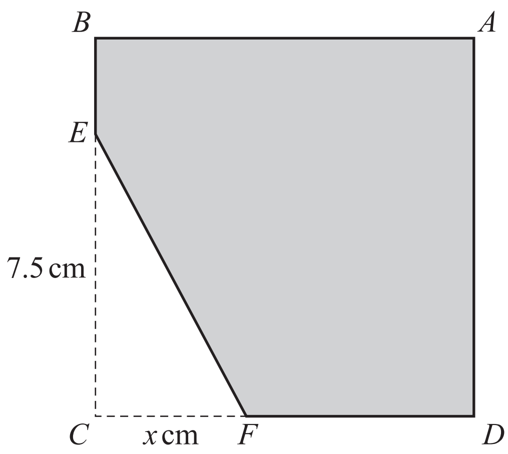
(b) Find the speed of P when the spring first returns to its natural length. [4]

4



A uniform square lamina $ABCD$ has sides of length 10 cm. The point E is on BC with $EC = 7.5$ cm, and the point F is on DC with $CF = x$ cm. The triangle EFC is removed from $ABCD$ (see diagram). The centre of mass of the resulting shape $ABEFD$ is a distance $\bar{x}$ cm from CB and a distance $\bar{y}$ cm from CD.

(a) Show that $\bar{x} = \dfrac{400 - x^2}{80 - 3x}$ and find a corresponding expression for $\bar{y}$. [4]

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The shape *ABEFD* is in equilibrium in a vertical plane with the edge *DF* resting on a smooth horizontal surface.

(b) Find the greatest possible value of x, giving your answer in the form $a + b\sqrt{2}$, where a and b are constants to be determined. [3]

5 A particle P is moving along a straight line with acceleration $3ku - kv$ where v is its velocity at time t, u is its initial velocity and k is a constant. The velocity and acceleration of P are both in the direction of increasing displacement from the initial position.

(a) Find the time taken for P to achieve a velocity of $2u$. [3]

(b) Find an expression for the displacement of P from its initial position when its velocity is $2u$. [5]

6 A particle P of mass m is moving with speed u on a fixed smooth horizontal surface. The particle strikes a fixed vertical barrier. At the instant of impact the direction of motion of P makes an angle α with the barrier. The coefficient of restitution between P and the barrier is e. As a result of the impact, the direction of motion of P is turned through 90°.

(a) Show that $\tan^2\alpha = \dfrac{1}{e}$. [3]

The particle P loses two-thirds of its kinetic energy in the impact.

(b) Find the value of α and the value of e. [5]

7 A hollow cylinder of radius a is fixed with its axis horizontal. A particle P, of mass m, moves in part of a vertical circle of radius a and centre O on the smooth inner surface of the cylinder. The speed of P when it is at the lowest point A of its motion is $\sqrt{\frac{7}{2}ga}$.

The particle P loses contact with the surface of the cylinder when OP makes an angle θ with the upward vertical through O.

(a) Show that $\theta = 60°$. [5]

(b) Show that in its subsequent motion P strikes the cylinder at the point A. [5]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

Cambridge Assessment International Education

Cambridge International AS & A Level

FURTHER MATHEMATICS **9231/32**

Paper 3 Further Mechanics 3 **May/June 2020**

MARK SCHEME

Maximum Mark: 50

Published

Students did not sit exam papers in the June 2020 series due to the Covid-19 global pandemic.

This mark scheme is published to support teachers and students and should be read together with the question paper. It shows the requirements of the exam. The answer column of the mark scheme shows the proposed basis on which Examiners would award marks for this exam. Where appropriate, this column also provides the most likely acceptable alternative responses expected from students. Examiners usually review the mark scheme after they have seen student responses and update the mark scheme if appropriate. In the June series, Examiners were unable to consider the acceptability of alternative responses, as there were no student responses to consider.

Mark schemes should usually be read together with the Principal Examiner Report for Teachers. However, because students did not sit exam papers, there is no Principal Examiner Report for Teachers for the June 2020 series.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the June 2020 series for most Cambridge IGCSE™ and Cambridge International A & AS Level components, and some Cambridge O Level components.

This document consists of **14** printed pages.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:
Marks must be awarded in line with:
- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:
Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:
Marks must be awarded **positively**:
- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
Rules must be applied consistently e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:
Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

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B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

Abbreviations

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SOI	Seen Or Implied
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WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks
1	For greatest height, $T = \dfrac{u}{2g}$	B1
	At $t = \dfrac{2T}{3}, \uparrow v_v = \dfrac{u}{2} - \dfrac{2Tg}{3} = \dfrac{u}{6}$	M1
	$\rightarrow v_h = \dfrac{u\sqrt{3}}{2}$	A1
	Speed = $\sqrt{v_v^{\ 2} + v_h^{\ 2}} = \sqrt{\dfrac{u^2}{36} + \dfrac{3u^2}{4}}$	M1
	$= \dfrac{\sqrt{7}}{3}u$	A1
		5

Question	Answer	Marks
2	For A: $T = 3mg$ For B: $\uparrow$ $T\cos\theta = mg$	M1
	Equate: $3mg\cos\theta = mg$ $\cos\theta = \frac{1}{3}$	A1
	$\rightarrow T\sin\theta = mr\omega^2$ with $r = (a-x)\sin\theta$	M1
	Equate: $3mg = m(a-x)\omega^2$	A1
	$x = \frac{a}{4}$	A1
		5

Question	Answer	Marks
3(a)	$T - mg = m.a$	M1
	$T = 5mg.\frac{1}{2}a / a \ = \frac{5}{2}mg$	M1
	$a = \frac{3}{2}g$ (upwards) **AG**	A1
		3

Question	Answer	Marks
3(b)	Gain in KE = $\frac{1}{2}mv^2$ Gain in GPE = $\frac{1}{2}mga$	B1
	Loss in EPE = $\frac{1}{2}\frac{5mg.\left(\frac{1}{2}a\right)^2}{a}$	B1
	$\frac{1}{2}mv^2+\frac{1}{2}mga=\frac{1}{2}\frac{5mg.\left(\frac{1}{2}a\right)^2}{a}$ $[\Rightarrow \frac{1}{2}mv^2+\frac{1}{2}mga=\frac{5}{8}mga\ ]$	M1
	$v=\frac{1}{2}\sqrt{ga}$	A1
		4

Question	Answer	Marks
4(a)	<table><tr><td></td><td>Area</td><td>Centre of mass from *BC*</td><td>Centre of mass from *DC*</td></tr><tr><td>Square</td><td>100</td><td>5</td><td>5</td></tr><tr><td>Triangle</td><td>½ *x*. 15/2</td><td>$\frac{1}{3}x$</td><td>$\frac{5}{2}$</td></tr><tr><td>Shape *ABEFD*</td><td>$100-\frac{15}{4}x$</td><td>$\bar{x}$</td><td>$\bar{y}$</td></tr></table> Take moments about *BC*: $\left(100-\frac{15}{4}x\right)\sigma\,.\,\bar{x}=500\sigma-\frac{15}{4}x\sigma.\frac{1}{3}x$ (**M1** for all terms present)	**M1**
	$\bar{x}=\frac{400-x^2}{80-3x}$ **AG**	**A1**
	Take moments about *DC*: $\left(100-\frac{15}{4}x\right).\ \bar{y}\ =100\times 5-\frac{15}{4}x.\frac{5}{2}$	**M1**
	$\bar{y}\ =\frac{800-15x}{160-6x}$	**A1**
		4
4(b)	Use condition: $\bar{x}\geqslant x$	**B1**
	$2x^2-80x+400\geqslant 0$	**M1**
	$x=20-10\sqrt{2}$	**A1**
		3

Question	Answer	Marks
5(a)	$\dfrac{\mathrm{d}v}{3u-v}=k\mathrm{d}t$	M1
	$-\ln(3u-v)=kt+d$ $t=0, v=u:\quad d=-\ln 2u$	M1
	$v=2u:\quad t=\dfrac{1}{k}\ln 2$	A1
		3
5(b)	$v\dfrac{\mathrm{d}v}{\mathrm{d}x}=3ku-kv\ \left[\Rightarrow\ \dfrac{v\mathrm{d}v}{3u-v}=k\mathrm{d}x\right]$	B1
	$\dfrac{\left(-(3u-v)+3u\right)\mathrm{d}v}{3u-v}=k\mathrm{d}x$ so $-v-3u\ln(3u-v)=kx+c$	M1A1
	$x=0, v=u:\ c=-u-3u\ln 2u$	M1
	$v=2u:\ x=\dfrac{u}{k}(3\ln 2-1)$	A1
		5

Question	Answer	Marks
6(a)	Let components of velocity (parallel to plane and perpendicular) after impact be (x, y) $y = v\cos\alpha = eu\sin\alpha$	B1
	$x = v\sin\alpha = u\cos\alpha$	B1
	Divide: $\tan\alpha = \dfrac{1}{e\tan\alpha}$: $\tan^2\alpha = \dfrac{1}{e}$.	B1
		3

Question	Answer	Marks
6(b)	$v^2 = \frac{1}{3}u^2$	B1
	$\left(\frac{u\cos\alpha}{\sin\alpha}\right)^2 = \frac{1}{3}u^2$	M1
	$(\tan\alpha)^2 = 3$	M1
	$\alpha = 60°$	A1
	$e = \frac{1}{3}$	A1
	Alternative method for **6(b)**	
	KE after impact $= \frac{1}{2}m\left(x^2 + y^2\right) = \frac{1}{2}m\left((u\cos\alpha)^2 + e^2(u\sin\alpha)^2\right)$	M1
	From **(a)** $\sin\alpha = 1/\sqrt{(1+e)}$ and $\cos\alpha = \sqrt{e}/\sqrt{(1+e)}$	B1
	KE $= \frac{1}{2}mu^2\left(\frac{e}{1+e} + \frac{e^2}{1+e}\right) = \frac{1}{2}mu^2e$	A1
	This is equal to $\frac{1}{3}\times\frac{1}{2}mu^2$ so $e = \frac{1}{3}$	M1
	$\tan\alpha = \sqrt{3},\ \alpha = 60°$	A1
		5

Question	Answer	Marks
7(a)	$(N+)mg\cos\theta = \dfrac{mv^2}{a}$	B1
	$\dfrac{1}{2}mv^2 - \dfrac{1}{2}m\dfrac{7ag}{2} = -mg(a + a\cos\theta)$	M1A1
	Loses contact when $N = 0$, so combine and simplify	M1
	$\cos\theta = \dfrac{1}{2}$: $\theta = 60°$ **AG**	A1
		5

Question	Answer	Marks
7(b)	When P is vertically below O, its horizontal displacement is $a\sin 60$, so time $T = \dfrac{a\sin 60}{v\cos 60} = a\sqrt{3}/v = \sqrt{\dfrac{6a}{g}}$	M1
	From **(a)**, $v^2 = \dfrac{1}{2}ag$	A1
	Vert: $h = \dfrac{v\sqrt{3}}{2}T - \dfrac{1}{2}g.T^2$	M1
	$\dfrac{3}{2}a - 3a = -\dfrac{3a}{2}$	A1
	This corresponds to the point A	A1
	Alternative method for question 7(b)	
	$y = x\sqrt{3} - \dfrac{4x^2}{a}$	M1A1
	Coordinates of A: $x = \dfrac{1}{2}a\sqrt{3},\ y = -\dfrac{3}{2}a$	B1
	Substitute coordinates into $y = x\sqrt{3} - \dfrac{4x^2}{a}$ and show that these satisfy this equation	M1A1
		5

Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME			
CENTRE NUMBER		CANDIDATE NUMBER	

8623401314

FURTHER MATHEMATICS **9231/33**

Paper 3 Further Mechanics **May/June 2020**

1 hour 30 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
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- You should use a calculator where appropriate.
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- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.
- Where a numerical value for the acceleration due to gravity (g) is needed, use $10\,\text{m}\,\text{s}^{-2}$.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Blank pages are indicated.

DC (PQ/FC) 187881/2

1 A particle P of mass m is attached to one end of a light inextensible string of length a. The other end of the string is attached to a fixed point O on a smooth horizontal plane. The particle P moves in horizontal circles about O. The tension in the string is $4mg$.

Find, in terms of a and g, the time that P takes to make one complete revolution. [2]

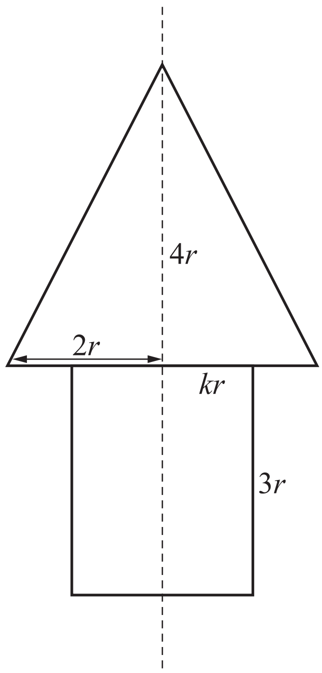
2 A particle Q of mass m kg falls from rest under gravity. The motion of Q is resisted by a force of magnitude mkv N, where $v\,\mathrm{m\,s^{-1}}$ is the speed of Q at time t s and k is a positive constant.

Find an expression for v in terms of g, k and t. [6]

3 A particle Q of mass m is attached to a fixed point O by a light inextensible string of length a. The particle moves in complete vertical circles about O. The points A and B are on the path of Q with AB a diameter of the circle. OA makes an angle of 60° with the downward vertical through O and OB makes an angle of 60° with the upward vertical through O. The speed of Q when it is at A is $2\sqrt{ag}$.

Given that T_A and T_B are the tensions in the string at A and B respectively, find the ratio $T_A : T_B$. [6]

4



A uniform solid circular cone, of vertical height $4r$ and radius $2r$, is attached to a uniform solid cylinder, of height $3r$ and radius kr, where k is a constant less than 2. The base of the cone is joined to one of the circular faces of the cylinder so that the axes of symmetry of the two solids coincide (see diagram). The cone and the cylinder are made of the same material.

(a) Show that the distance of the centre of mass of the combined solid from the vertex of the cone is $\dfrac{(99k^2+96)r}{18k^2+32}$. [4]

The point C is on the circumference of the base of the cone. When the combined solid is freely suspended from C and hanging in equilibrium, the diameter through C makes an angle α with the downward vertical, where $\tan\alpha = \frac{1}{8}$.

(b) Given that the centre of mass of the combined solid is within the cylinder, find the value of k. [4]

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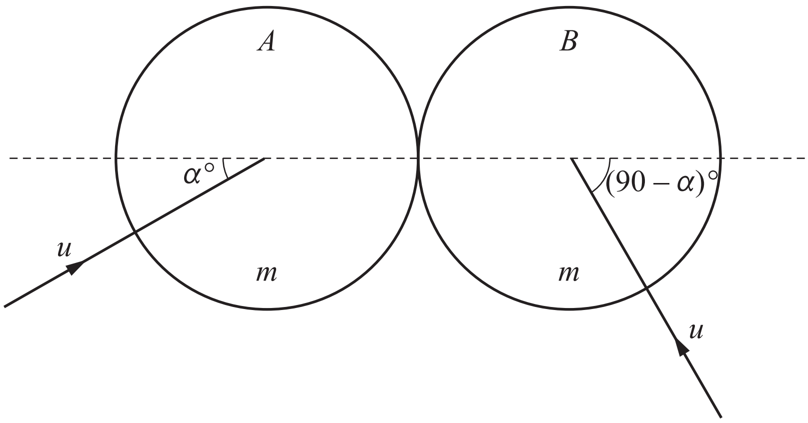

Two uniform smooth spheres A and B of equal radii each have mass m. The two spheres are each moving with speed u on a horizontal surface when they collide. Immediately before the collision A's direction of motion makes an angle of $\alpha°$ with the line of centres, and B's direction of motion is perpendicular to that of A (see diagram). The coefficient of restitution between the spheres is e.

Immediately after the collision, B moves in a direction at right angles to the line of centres.

(a) Show that $\tan\alpha = \dfrac{1+e}{1-e}$. [4]

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(b) Given that $\tan\alpha = 2$, find the speed of A after the collision. [4]

6 A particle P is projected with speed u at an angle θ above the horizontal from a point O on a horizontal plane and moves freely under gravity. The direction of motion of P makes an angle α above the horizontal when P first reaches three-quarters of its greatest height.

(a) Show that $\tan\alpha = \frac{1}{2}\tan\theta$. [6]

(b) Given that $\tan\theta = \frac{4}{3}$, find the horizontal distance travelled by P when it first reaches three-quarters of its greatest height. Give your answer in terms of u and g. [4]

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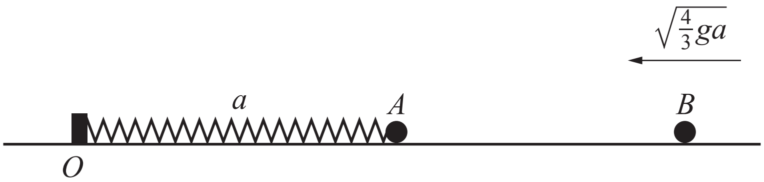

One end of a light spring of natural length a and modulus of elasticity $4mg$ is attached to a fixed point O. The other end of the spring is attached to a particle A of mass km, where k is a constant. Initially the spring lies at rest on a smooth horizontal surface and has length a. A second particle B, of mass m, is moving towards A with speed $\sqrt{\frac{4}{3}ga}$ along the line of the spring from the opposite direction to O (see diagram).

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(b) Find the value of μ. [4]

Additional Page

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8623401314

FURTHER MATHEMATICS **9231/33**

Paper 3 Further Mechanics **May/June 2020**

1 hour 30 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

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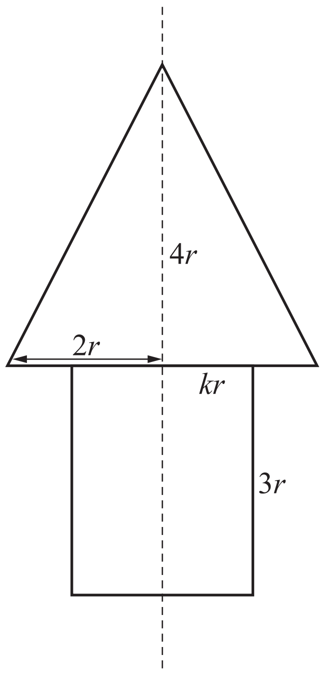

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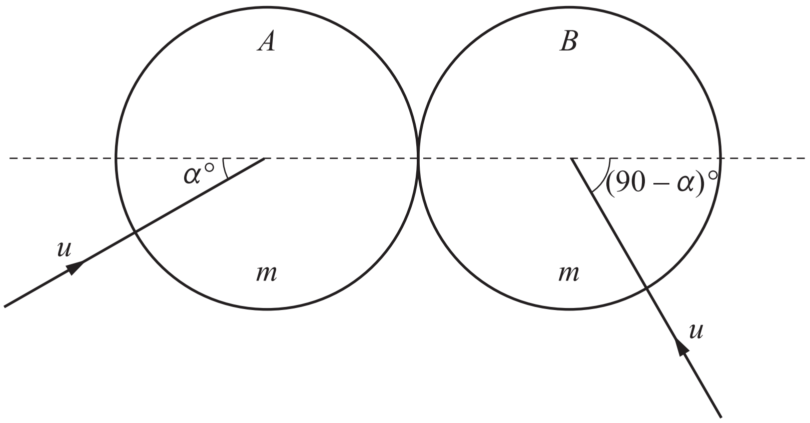

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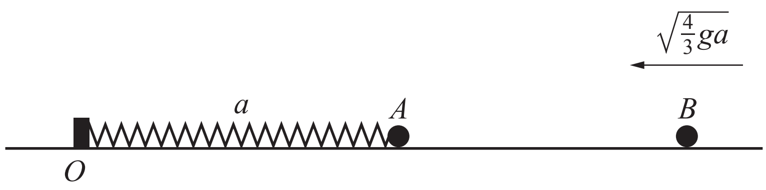

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